

# Stochastic Production Planning with Flexible Manufacturing Systems and Uncertain Demand: A Column Generation-based Approach

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**Abstract:** The ongoing pandemic, namely COVID-19, has rendered widespread economic disorder. The deficiencies have delayed production at manufacturers in several industries on the supply side. The effects of disruption were more notable for industries with longer supply chains, especially reaching East Asia. Regarding the demand, sectors can be divided into three categories: i) the ones, like e-commerce companies, that experienced augmented demand; ii) the ones with a plunged demand, like what hotels and restaurants experience; iii) the companies experiencing a roller-coaster-ride business. After mitigation efforts, the economy started recovering, resulting in increased demand. However, regardless of their struggles, the companies have not fully returned to their pre-pandemic levels. One of the strategies to gain resilience in its supply chain and manage the disruptions is to employ flexible/hybrid manufacturing systems. This paper considers a flexible/hybrid manufacturing production setting with typically dedicated machinery to satisfy regular demand and a *flexible manufacturing system* (FMS) to handle surge demand. We model the uncertainty in demand using a scenario-based approach and allow the business to make here-and-now and wait-and-see decisions exploiting the cost-effectiveness of the standard production and responsiveness of the FMS. We propose a column generation-based algorithm as the solution approach. Our computational analysis shows that this hybrid production setting provides highly robust response to the uncertainty in demand, even with high fluctuations.

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## 1. INTRODUCTION

The COVID-19 pandemic caused havoc in all sectors worldwide. Healthcare, hospitality, and tourism were among the most affected, though almost all businesses faced inevitable disruptions. People's behavior has dramatically changed due to concerns and restrictions taking place, and this, in turn, caused several transformations for the business, employees, and consumers. Some of these transformations are relatively positive for certain businesses/people, such as e-commerce activities and customer satisfaction have increased, opportunities for working remotely arises, universities and other institutions have become more proficient in online education, less petroleum usage reduced the oil prices and carbon emissions worldwide, etc. Nevertheless, others were quite negative. Shut-downs and travel restrictions caused many businesses to struggle or even fail. Raw material deficiencies have delayed production at factories in several industries on the supply side, and so this resulted in rising input prices in subsequent industries. People lost their jobs due to the

pandemic or had to be furloughed during lockdown phases Fletcher et al. (2021).

On the supply chain side, the effects of disruption were more notable for industries with longer supply chains, especially reaching East Asia. The supply chain disruptions almost immediately started as the outbreak took over China in early 2020. The supply shock was then followed by extreme demand fluctuations globally. Regarding the demand, sectors can be divided into three categories: i) the ones, like e-commerce companies, that experienced augmented demand; ii) the ones with a plunged demand, like what hotels and restaurants experience; iii) the companies experiencing a roller-coaster-ride business. In any category, however, the consumers demand low prices as the pandemic took a toll on their household incomes, whereas businesses cannot survive for long without hiking the prices under such a disrupted supply environment.

Businesses have tried to adapt to the “new normal” since the early stages of the pandemic and started employing mitigation strategies to overcome these supply shocks. Some firms switched to local suppliers in an attempt to

shorten the supply lines as all key aspects of a supply chain, such as transportation, storage, and currency exchange rates, are affected by the pandemic. Multisourcing was another option to secure the supply by hedging the potential risks among multiple suppliers. Demand planning/smoothing was employed to reduce the peak demand, which gives the firm an edge to handle surge demand. Also, to deal with uncertainties in supply/demand matching, some firms built up extra inventories to sustain their business through prolonged periods of disruption. Here, the challenge for the firms was to make their supply chains more resilient without a toll on their competitiveness.

One of the strategies to gain resilience in its supply chain and manage the disruptions is to employ flexible/hybrid manufacturing systems. To this end, compared to its alternatives, having flexibility in manufacturing might be more cost-effective as the other options have certain downsides such as increased labor costs in local production, increased holding costs in keeping an extra inventory, and so on. As customer expectations are ever-increasing, manufacturing techniques are also evolving to keep up with those expectations. The concept of mass customization, manufacturing a highly varied product portfolio in a mass production scheme, forces the companies to seek manufacturing systems that are both flexible and can produce high-quality items at a meager cost. This, in turn, highlights the significance of designing flexible manufacturing systems (FMSs) that are capable of producing a variety of goods belonging to a specific class. However, the downside of the FMSs is that their initial investment is expensive, and they typically require high maintenance times.

According to Groover (2007) the idea of FMS was first proposed by David Williamson in 1960s under the name 'System 24,' which stands for a machine that operate for 24 hours a day without any need for a human operator. However, the definition of FMS, which is widely accepted nowadays, is proposed by Browne et al. (1984). KUSTAK (1985) present the structure of FMS, its process, and problem views. By defining the design and operational problems different aspects of flexibility is presented in this study. Several survey papers there are about FMS. We refer the readers to Yadav and Jayswal (2018), Yelles-Chaouche et al. (2021), and Yadav and Jayswal (2018) for the most recent and fundamental reviews of the papers about FMS.

The most significant advantage of FMSs is to provide versatility to the manufacturing environment in case of rapid changes. It mainly offers two types of flexibility: (i) machine flexibility which refers to the flexibility of machines to perform different operations, and (ii) routing flexibility which refers to the ability to change the order of operations on a part. Even though FMS's adaptability favors them in efforts to reduce response times, reduce delays, increase machine/labor productivity, and hence increase customer satisfaction, they require skilled/flexible labor, extensive initial planning, and possibly higher maintenance costs. Therefore, in most cases, it is necessary to exploit its potential benefits to the full extent; otherwise, the firms might face unnecessary inefficiencies/costs while trying to increase productivity.

Tang (2006) discuss robust supply chain strategies in the face of disruption and suggest that companies that deploy contingency plans would be less vulnerable in case of disruption. Tomlin (2006) consider single product procurement with two non-identical suppliers and compare different mitigation strategies such as inventory building, sourcing from the reliable high-cost supplier, and passive acceptance and discuss the optimality of these strategies under different conditions. Wu et al. (2007) suggest a network-based modeling methodology to understand how disruptions affect the supply chains, which will lead to developing better mitigation strategies. Tang et al. (2014) propose a series of models to study supply disruptions under different scenarios with deterministic and stochastic demand settings. Fahimnia et al. (2015) provide a thorough review of the models to deal with potential supply chain risks. Mejía and Lefebvre (2020) consider FMS with operation interruptions and unreliable resources and propose an anytime graph search algorithm with an objective function that encompasses performance and risk.

Studying the literature carefully, in turn, highlights the significance of designing FMSs that are capable of producing a variety of goods belonging to a specific class. In this study, we consider multiple companies trying to satisfy the demand for multiple products using a common production environment. This common production floor consists of two different machines; (i) *typical manufacturing machines* (TMM) and (ii) a limited number of FMS. A product can be produced on a TMM or an FMS. The production cost and the amount of the product that can be produced on each of them are different. This two-mode setup provides the efficiency of traditional mass manufacturing under stationary demand patterns with a potentially lower cost of production per unit and flexibility under highly volatile demand patterns with a potentially higher cost of production per unit but highly adjustable output.

*Contributions* of this study can be summarized in three-fold. Firstly, to the authors' best knowledge, this is the first study that proposes an optimization-under-uncertainty framework in flexible manufacturing systems literature; *stochastic production planning* (SPP) is the first problem that considers FMSs and TMMs in a manufacturing company and provides resilient solutions against the demand uncertainty. Secondly, we propose a *column generation* (CG) based heuristic approach that solves the problem for realistically sized instances for twenty different cases, while a commercial solver cannot even create an instance of the standard model. To evaluate the performance of the proposed algorithm, we provide an efficient lower bound mechanism for the problem that is obtained by solving the linear programming (LP) relaxation of proposed mixed-integer programming models with one FMS and one TMM. The numerical results show that the proposed CG approach provides (on average) less than a 3% optimality gap for the setting that adopts both FMS and TMM for realistically sized instances. Lastly, regardless of the difficulties due to uncertain demand, we develop effective schedules that improve the operational cost of standard production schemes that do not use FMS by (on average) 14%.

The remainder of the paper is organized as follows. Section 2 defines the problem by proposing the mathematical

model of the proposed problem. Section 3 provides the solution approaches. The numerical experiments are given in Section 4. In Section 5, we provide the conclusion of the study.

## 2. PROBLEM DEFINITION

In this section, we define the problem formally, list our assumptions, and present the mathematical formulation of SPP. In SPP, we consider a set of companies  $N = \{1, \dots, n\}$  that would like to cooperate in order to reduce their procurement costs. Each company is trying to fulfill the demand of multiple products ( $J = \{1, \dots, R\}$ ) over a finite planning horizon,  $T$ . Moreover, uncertainty in demand is modeled via a scenario tree approach. We consider two types of decision variables: here-and-now and wait-and-see decision variables. Here-and-now decision variables are decided at the beginning of the planning horizon using the typical machines, and wait-and-see decision variables are more situational/flexible and are satisfied utilizing FMSs, and their production is determined/adjusted according to the realizations of the demand in the planning horizon. It is assumed that both TMMs and FMSs are capable of producing all products, and there is a sequence-based setup time between each production. Units of product  $j$  that a TMM and an FMS can produce are denoted by  $a_j$  and  $a'_j$ , respectively. Demand of company  $n$  for product  $j$  in node  $e$  is denoted by  $d_{nej}$ . Each company meets the demand by ordering the product from a common supplier. The supplier may either manufacture or order from a third party to meet the demand of the companies. A fixed cost is incurred with each production/order, and a variable cost is incurred depending on the amount of production/order. The fixed production cost of producing product  $j$  for TMMs for period  $t$  is denoted by  $A_{tj}$ , while the unit variable cost for TMMs is denoted by  $v_{tj}$ . The fixed and variable costs for FMSs are denoted by  $A'_{tj}$  and  $v'_{tj}$ , respectively. When the product is ordered before the realization of the demand, the cost of keeping inventory is incurred up until the actual time of the demand, and the cost of keeping a unit product in inventory in each period for product  $j$  is denoted by  $h_j$ . The supplier has a certain production capacity for each period, and this production capacity is indicated by  $C_j$ . The supplier also has a certain inventory capacity for storing items after each period, and this inventory capacity is indicated by  $S$ . The demand of the grand coalition, where all the companies come together, in period  $t$  is indicated by  $d_{nej}$ . Finally, we assume that the unit outsourcing cost of product  $j$  in each period is denoted by  $\beta_j$ . The nomenclature for the SPP is presented in Table 1.

In Figure 1, we represent a general scenario tree with the notations used in SPP. This figure provides a tangible representation of the demand tree structure. For example, the immediate predecessor of node  $e_6$  is node  $e_2 = \text{pd}(e_6, 1)$ . Also, set of children of node  $e_1$  in the next period is presented by  $\bar{N}(e_1, 1)$  and for the next two periods is presented by  $\bar{N}(e_1, 2)$ . Each node  $e \in N$  in the scenario tree denotes a demand realization at period  $t(e)$  with its associated probability  $\text{pr}(e)$ , and each path from the root node (0) to a leaf node  $e \in N$  is referred to as a *scenario* of the tree, i.e., denoted by  $\text{pd}(e_{14}) = \{0, e_2, e_6, e_{14}\}$ , where

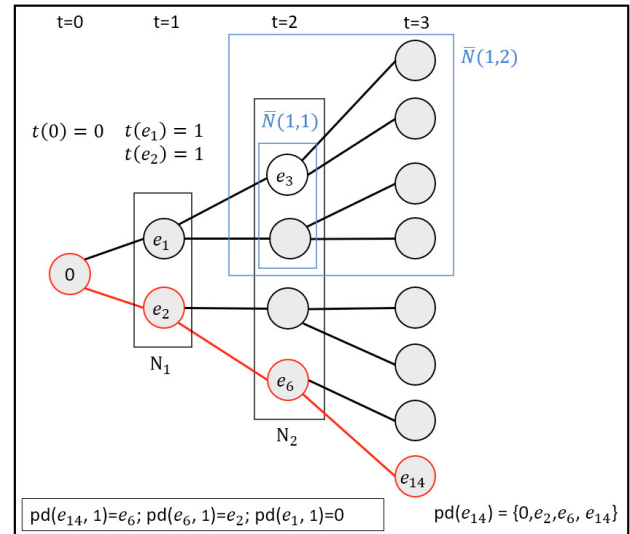


Fig. 1. Problem structure of SPP

$N_T$  denotes the set of nodes in the last period of the planning horizon.

SPP can be formulated considering both TMM and FMS (MIP). It should be noted that in MIP, production decisions are divided into two groups, namely, here-and-now ( $\mathbf{y}$ ) that is done by TMMs and are scheduled at the beginning of the planning horizon and are independent of the scenario realizations, and scenario-specific wait-and-see ( $\mathbf{x}$ ) decisions that are done by FMSs.

MIP:

$$\min \sum_{e \in N_L} \text{pr}(e) \sum_{e' \in \text{pd}(e)} \left( \sum_j \sum_{k'} v'_{t(e')j} a'_{jk'} x_{k'e'j} + \sum_j \sum_{k'} A'_{t(e')j} x_{k'e'j} + \sum_j (h_j I_{e'j} + \beta_j B_{e'j}) \right) \quad (1)$$

$$+ \sum_t \sum_j \sum_k \left( v_{tj} a_j y_{ktj} + A_{tj} y_{ktj} \right)$$

$$\text{s.t. } (R-1)x_{k'e'j} + \sum_{j'} \sum_{e' \in \bar{N}(e, s_{jj'}-1)} x_{k'e'j'} \leq R, \forall j, e, k' \quad (2)$$

$$(R-1)y_{ktj} + \sum_{j'} \sum_{t'=t}^{t+s_{jj'}-1} y_{kt'j'} \leq R, \forall j, k, t \quad (3)$$

$$\sum_j \sum_k \sum_t a_j y_{ktj} \leq C \quad (4)$$

$$\sum_j \sum_e \sum_{k'} a'_{jk'} x_{k'e'j} \leq C' \quad (5)$$

$$\sum_{e \in N_t} \sum_j I_{e'j} \leq S, \forall t \quad (6)$$

$$\sum_{e \in N_t} B_{e'j} \leq \mu_j, \forall t, j \quad (7)$$

$$B_{\text{pd}(e,1)j} - I_{e'j} = I_{\text{pd}(e,1)j} + \sum_k a_j y_{kt(e)j} + \sum_{k'} a'_{jk'} x_{k'e'j} - \sum_n d_{nej} + B_{e'j}, \forall e, j \quad (8)$$

$$x_{k'e'j} \in \{0, 1\}; y_{ktj} \in \{0, 1\}, \forall e, t, j, k, k' \quad (9)$$

$$I_{e'j}, B_{e'j} \geq 0, \forall e, j \quad (10)$$

Table 1. Nomenclature for SPP

<b>Sets</b>	
$N$	set of companies ( $\{1, 2, \dots, W\}$ ) – indexed by $n$
$J$	set of products ( $\{1, 2, \dots, R\}$ ) – indexed by $j$
$K$	set of TMMs ( $\{1, 2, \dots, M\}$ ) – indexed by $k$
$K'$	set of FMSs ( $\{1, 2, \dots, M'\}$ ) – indexed by $k'$
$\bar{K}$	set of machines ( $\bar{K} = K \cup K'$ ) – indexed by $\bar{k}$
$T$	set of time periods in the planning horizon ( $\{1, 2, \dots, L\}$ ) – indexed by $t$
$E$	set of demand scenario or node indexes ( $\{1, 2, \dots, S\}$ ) – indexed by $e$
$N_t$	set of nodes in period $t \in T$
$\bar{N}(e, l)$	set of children nodes of $e$ in the next $l$ periods $\cup \{e\}$
<b>Parameters</b>	
$a_j$	units of product $j$ that each TMM produces
$a'_j$	units of product $j$ that each FMS produces
$s_{jj'}$	setup time between product $j$ and $j'$ on TMM
$s'_{jj'}$	setup time between product $j$ and $j'$ on FMS
$d_{nej}$	demand of company $n$ for product $j$ in node $e$
$v_{tj}$	variable cost of production of product $j$ in period $t$ by a TMM
$v'_{tj}$	variable cost of production of product $j$ in period $t$ by an FMS
$A_{tj}$	fixed cost of production of product $j$ in period $t$ by a TMM
$A'_{tj}$	fixed cost of production of product $j$ in period $t$ by an FMS
$h_j$	unit holding cost of product $j$
$\beta_j$	unit outsourcing cost of product $j$
$C$	production capacity of TMM during the planning horizon
$C'$	production capacity of FMS during the planning horizon
$S$	inventory capacity per period
$R$	a very large number
$\text{pd}(e, 1)$	immediate predecessor of node $e$
$\text{pd}(e)$	nodes in the path from the source node to $e$
$\text{pr}(e)$	probability of scenarios in path $\text{pd}(e)$ being realized
$t(e)$	period of node $e$
<b>Decision variables</b>	
$y_{ktj}$	1 if there is a production of TMM for product $j$ on machine $k$ in period $t$ & 0 otherwise
$x_{k'e_j}$	1 if there is a production of FMS for product $j$ on machine $k'$ in node $e$ & 0 otherwise
$I_{ej}$	inventory of product $j$ at the end of node $e$
$P_{ej}$	amount of product $j$ outsourced at the end of node $e$

In this model, the objective (1) is to minimize the sum of the expected production that is done by TMM and FMS, inventory, and outsourcing costs. Constraints (2) enforce the setup times of FMSs, and constraints (3) enforce the setup times of TMMs. The total production capacity of TMMs and FMSs are enforced by constraints (4) and (5), respectively. While constraints (6) ensure the total inventory capacity for each period, the total amount that can be outsourced for each product in each period is ensured by constraints (7). Constraints (8) ensure the inventory balance for each product and the time period overall scenarios. Constraints (9) and (10) are binary and non-negativity constraints, respectively.

### 3. SOLUTION APPROACH

Solving MIP is not an easy task for large instances due to the size of the problem and long planning horizons. In this section, we propose a column generation-based heuristic algorithm. In order to do so, we decompose the mathematical model using the Dantzig Wolfe decomposition. This algorithm starts with solving the LP-relaxation of *restricted master problem* (RMP), in which there are two types of columns, each representing the production schedule of TMMs and FMSs. Following that, after obtaining the dual variables from the RMP, corresponding subproblems (PP-y and PP-x) are solved, and profitable columns are added to the RMP. These steps are continued until all profitable columns are added to RMP or the time limit has been reached. In the final step of the algorithm, by considering all the generated columns, we solve RMP

with integer decision variables with TMMs and FMSs decision variables. It must be noted that since in the final step of the algorithms, we solve RMP with integer decision variables instead of branching on those decision variables, which would take a great amount of time, these algorithms are different from orthodox branch-and-price algorithms, and that is why we call them heuristics. For detailed explanation of column generation algorithm, we refer the readers to Lübbecke and Desrosiers (2005), Desaulniers et al. (2005) and Lübbecke (2010).

The column generation-based algorithm we propose here follows a sequential order. Notice that in RMP, this sequential column generation approach shall continue until the objective function values of the subproblems PP-x and PP-y are non-negative, i.e., there does not exist any profitable column that may improve the current solution of RMP. Finally, RMP is solved as an integer programming problem whose solution becomes the final solution of SPP.

The *reduced master problem* RMP is given as follows where decision variable  $y_l$  represents the TMM productions which follow here-and-now schedule  $l$ , and  $x_m$  represents the FMS productions that follow wait-and-see schedule  $m$ ,  $\tilde{c}_l$  is the total donation cost of TMM schedule  $l$  while  $\hat{c}_m$  is the total donation cost of FMS schedule  $m$ . Moreover,  $\theta_{jt(e)}^l$  is the total number of product  $j$  that is produced in time period  $t(e)$  by production schedule  $l$  of a TMM, and  $\lambda_{ej}^m$  is the total number product  $j$  that is produced in node  $e$  if an FMS follows schedule  $m$ .

**RMP:**

$$\min_{I, P} \sum_{e \in N_L} \text{pr}(e) \left( \sum_{e' \in \text{pd}(e)} \sum_j h_j I_{e'j} + \beta_j B_{e'j} \right) + \sum_m \hat{c}_m x_m + \sum_l \tilde{c}_l y_l \quad (11)$$

$$\text{s.t.} \quad \sum_{e \in N_t} \sum_j I_{ej} \leq S, \forall t \quad (12)$$

$$\sum_{e \in N_t} B_{ej} \leq \mu_j, \forall t, j \quad (13)$$

$$B_{\text{pd}(e,1)j} + I_{ej} = I_{\text{pd}(e,1)j} - \sum_n d_{nej} + B_{ej} + \sum_m \lambda_{ej}^m x_m + \sum_l \theta_{t(e)j}^l y_l, \forall e, j \quad (14)$$

$$\sum_j \sum_m \sum_e \lambda_{ej}^m x_m \leq C \quad (15)$$

$$\sum_j \sum_l \sum_t \theta_{t(e)j}^l y_l \leq C' \quad (16)$$

$$\sum_l y_l \leq M \quad (17)$$

$$\sum_l x_m \leq M' \quad (18)$$

$$y_l \geq 0, x_m \geq 0, \forall l, m \quad (19)$$

$$I_{ej}, B_{ej} \geq 0, \forall e, j \quad (20)$$

In RMP, the objective (11) is to minimize the total (expected) cost of the columns, including the sum of productions by TMM and FMS, inventory holding, and outsourcing. Constraints (12) and (13) ensure the inventory and outsourcing capacities, respectively. Constraints (14) ensure the inventory balance. Production capacities of FMSs and TMMs are enforced by constraints (15) and (16), respectively. While constraints (17) enforce that the total TMM columns must be less than or equal to the number of TMMs, constraints (18) ensure that the total FMS columns must be less than or equal to the number of FMSs. It should be noted that the dual variables obtained from (10) - (16) are denoted by  $\delta_t$ ,  $\gamma_{tj}$ ,  $\eta_{ej}$ ,  $\iota$ , and  $\zeta$ , respectively. The sub-problems that must be solved for TMMs and FMSs are denoted by PP-y and PP-x, respectively. *Subproblem* PP-y is provided below:

**PP-y:**

$$\min_q \sum_t \sum_j q_{jt} (a_j v_{tj} + A_{tj} - \sum_{e \in N_t} a_j \eta_{ej}) - \sum_t \delta_t - \iota \quad (21)$$

$$\text{s.t.} \quad (M-1)q_{jt} + \sum_{j'} \sum_{t'=t}^{t+s_{jj'}-1} q_{j't'} \leq M, \forall j, t \quad (22)$$

$$q_{jt} \in \{0, 1\}, \forall j, t \quad (23)$$

In PP-y, the objective function (21) is minimizing the total production cost of a TMM, and constraints (22) enforce the setup times between two consecutive productions of a TMM. After solving PP-y, the following calculations must be done:  $\theta_{tj}^l = a_j q_{tj}$  and  $\tilde{c}_l = \sum_j \sum_t q_{tj} (a_j v_{tj} + A_t)$ . *Subproblem* PP-x is given as follows:

**PP-x**

$$\min_z \sum_e \text{pr}(e) \sum_j (z_{ej} (a'_j v'_{t(e)j} + A'_{t(e)j})) \quad (24)$$

$$- \left( \sum_j \sum_e a'_j z_{ej} \eta_{ej} \right) - \sum_t \delta_t - \iota$$

$$\text{s.t.} \quad (M-1)z_{ej} + \sum_{j'} \sum_{e' \in \bar{N}(e, s_{jj'}-1)} z_{e'j'} \leq M, \forall e, j \quad (25)$$

$$z_{ej} \in \{0, 1\}, \forall e, j \quad (26)$$

In PP-X, the objective function (24) is minimizing the total production cost of an FMS, and constraints (25) enforce the setup times between two consecutive production of an FMS. After solving PP-x, we have the following calculations:  $\lambda_{ej}^m = a'_j z_{je}$  and  $\hat{c}_l = \sum_e \text{pr}(e) \sum_j z_{ej} (a'_j v'_{t(e)j} + A'_{t(e)j})$ .

#### 4. COMPUTATIONAL EXPERIMENTS

In this section, we present the results of a numerical study conducted based on random instances motivated by real-life data. Our numerical study aims to evaluate the performance of the proposed method in terms of solution quality. The computational experiments are carried out on a 64-bit Windows Server with two 2.4 GHz Intel Xeon CPUs and 24 GB RAM. The algorithms are implemented using Python Programming Language and GUROBI Solver version 9.1.1. Throughout this section, a period corresponds to a month, and we solve the problem for a 12-month planning horizon, which is the largest dimension that can be solved in possible CPU times. The proposed mathematical model, namely, MIP, cannot be solved because of memory problems. Even for the column generation-based algorithm, there are many decision variables; therefore, we limit the CPU time of the proposed algorithm to 24 hours. In order to investigate the performance of the solutions of the column generation-based heuristics, we report the optimality gaps with respect to lower bounds. The lower bounds in this study are obtained by solving the *linear programming* (LP) relaxation of MIP. The optimality gap of wait-and-see decision making (using FMS) and the improvement of having wait-and-see production schedules over only here-and-now (TMMs) are presented in Table 2. In Table 2, the first column represents the instance number, the second column represents the optimality gap of the proposed algorithm w.r.t. LP-relaxation of MIP, and the last column represents the improvement in the objective function value of MIP when the wait-and-see schedules are included over utilizing only TMMs. Notice that the system benefits from the wait-and-see production plans because they yield scenario-based inventory management flexibility to the problem. On average, the proposed column generation-based heuristic provides only 2.68% gap w.r.t. the lower bound while providing 0.72% and 4.19% gaps for the best- and the worst-case instances, respectively. Moreover, including wait-and-see schedules decreases the total cost over here-and-now only schedules case by 13.41% on average, while the lowest and the greatest improvements are 6.00% and 21.03%, respectively, for the reported instances.

Table 2. Optimality gaps and improvements for here-and-now and wait-and-see production schedules

Instance	Gap	Improvement
1	3.31%	13.65%
2	3.03%	15.76%
3	3.19%	17.48%
4	2.96%	14.50%
5	3.32%	15.80%
6	3.97%	19.24%
7	0.73%	21.03%
8	2.90%	14.78%
9	4.13%	18.98%
10	3.81%	15.72%
11	3.07%	13.57%
12	1.93%	8.39%
13	1.57%	8.69%
14	0.72%	8.90%
15	4.19%	6.00%
16	2.10%	11.78%
17	1.78%	10.87%
18	2.32%	10.76%
19	2.17%	10.35%
20	2.37%	11.99%
Average	2.68%	13.41%
Best	0.72%	21.03%
Worst	4.19%	6.00%

## 5. CONCLUSION

After the COVID-19 outbreak, many companies have been forced to deal with demand disruptions. Some companies built up extra inventories to gain resilience against prolonged periods of disruption. One of the strategies to gain resilience in its supply chain and manage the disruptions is to employ flexible/hybrid manufacturing systems. To this end, in this study, we propose *stochastic production planning* (SPP) problem that takes into account the uncertain demand of products in hybrid manufacturing systems that takes advantage of a flexible manufacturing system (FMS) and aims to minimize the total (expected) production, holding, and outsourcing costs. We model the uncertainty in demand using a scenario tree approach. SPP is the first optimization model that yields a hybrid scheme for producing by providing resilient solutions against demand uncertainty. Utilizing FMS and typical manufacturing machines (TMM) provides more flexible production schedules since, based on the realized values of demand over time, FMS adjust their production schedules. Moreover, we propose a *column generation* (CG) based heuristic that solves the problem in less than 24 hours for realistically sized instances while a commercial solver cannot even create an instance of the standard model. The numerical results show that the proposed CG approach yields less than 3% (average) optimality gap for realistically sized instances. Last but not least, we develop effective hybrid production schedules that improve the operational cost of a standard production scheme that uses only TMM by 14% on average.

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