Int. J. Production Economics 278 (2024) 109418



Contents lists available at ScienceDirect

International Journal of Production Economics

journal homepage: www.elsevier.com/locate/ijpe





A deterministic fluid model for production and energy mode control of a single machine

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ARTICLE INFO

Keywords:
Manufacturing
Optimal production and energy control
Energy efficiency
Economic Production Quantity (EPQ) model
Resource scheduling

ABSTRACT

Improving machines' energy efficiency through dynamic energy mode control to meet demand requirements with minimal energy consumption is a promising approach. This study considers a machine operating in working, idle, off, and warmup energy modes with different energy consumption in each mode. A deterministic fluid model is developed to analyze an energy mode control policy that determines when to keep the machine working, off or idle, and switch to other modes based on the inventory/backlog level to minimize the total energy, inventory, and backlog costs. This approach facilitates the derivation of closed-form expressions for the optimal thresholds and the associated costs. This modeling approach allows us to prove that a policy that operates the machine between the working and off modes or the working and idle modes is always better than a hybrid policy that operates the machine in working, off, and idle modes simultaneously. We use the solution of the deterministic fluid model to propose an approximate policy for machines with stochastic production, warmup, and demand processes. We compare the results of the proposed approximation method with the optimal solution of a stochastic system where the production and warmup times are exponential and the demand inter-arrival times have Erlang distribution. The optimal solution for the stochastic system is determined by solving a Markovian Decision Process (MDP). Our numerical experiments show that the proposed approximation method predicts the optimal policy type for the stochastic case with a 89.3% accuracy, and the average error between the optimal cost and the cost obtained with the approximation method is 1.37% for 729 different cases tested. Furthermore, the computational efficiency of the proposed approximation is around 250 times better than the effort to determine the optimal policy using an MDP approach. We propose this approximation method where the control parameters are given in closed form as an easy-to-implement and effective policy to control energy modes to minimize the total energy, inventory, and backlog costs. Furthermore, we present the deterministic fluid modeling approach as a versatile approach to analyze energy mode control problems.

1. Introduction

Manufacturing is one of the most significant contributors to the global energy consumption. Increasing concerns for the environmental effects of manufacturing coupled with growing energy costs led to a flurry of activities to improve energy efficiency in manufacturing. This further underscores the need for effective production planning and control across various time scales. Examples of energy control problems encountered in practice encompass a range of applications, including but not limited to automotive assembly lines (Tan et al., 2023b,a; Loffredo et al., 2023a), CNC machine lines (Squeo et al., 2019; Frigerio et al., 2020), and industrial part washers (Loffredo et al., 2023b). Inspired by a practical example of operating a car painting oven in automotive manufacturing, this study addresses the dynamic control of machine energy modes — including on, off, idle, and warmup states — throughout the day. When an oven is not engaged in drying, it can be switched to an idle mode, which consumes significantly less energy than its working mode. Similarly, when an oven is completely turned off, its energy consumption is minimized. However, reactivating the drying process following a shutdown involves a warmup phase, during which energy consumption is typically elevated compared to all other energy modes. This warmup period is necessary for the oven to attain the optimal operating temperature required for effective drying, resulting in higher energy consumption relative to its standard working condition.

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https://doi.org/10.1016/j.ijpe.2024.109418
Received 13 March 2024; Received in revised form 5 September 2024; Accepted 23 September 2024
Available online 30 September 2024
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Technological advancements, including manufacturing execution systems, Internet of Things (IoT) networks, and data analytics, combined with machinery capable of operating in multiple energy modes, have enabled the implementation of real-time, energy-aware production control policies in complex systems, as exemplified in the discussion above. Many of these energy-aware production control policies aim to reduce a resource's idle time by operating the resource in a less-energy-consuming mode (Gutowski et al., 2005; Mouzon et al., 2007). These control policies typically use the current system state information to control the energy mode of the machine, e.g., working, off, and idle dynamically. When these policies utilize buffer information, which encompasses quantities of waiting or completed products, they are classified as buffer-based policies. According to a buffer-based policy, a turned-off or idle machine is turned on when the buffer level decreases to a lower threshold and turned off or put into idle mode when an upper threshold is reached. In addition, there are also other policies using time information, such as the time since the last part arrived or since the previous turn-off of the machine, to control the machine, and these policies are referred to as time-based policies. The challenge in implementing these policies is typically finding the optimal values of the control parameters, which are the thresholds, to minimize the energy costs while meeting the production requirements.

Numerous studies in the literature have examined energy-mode control policies under the assumption of stochastic demand arrivals, production processes, and warmup times. These studies have typically utilized either simulation optimization or stochastic models to identify the optimal values of the control parameters. Such methods often exhibit significant computational complexity and require extensive computational resources. Furthermore, the resulting control policies and the underlying dynamics of these models tend to be challenging to implement in practical scenarios. They are often difficult for practitioners to comprehend and apply effectively. In contrast to these existing studies, our work introduces a straightforward deterministic fluid model to analyze buffer-based energy-mode control policies. The proposed approach facilitates the derivation of optimal thresholds and associated costs in closed form, thereby enhancing both the interpretability and practical applicability of the model and its results for practitioners. The closed-form expressions provide clear insights into how system parameters influence control parameters, enabling practitioners to make well-informed decisions with a precise understanding of the effects of parameter changes. These insights are not only valuable for optimizing operational efficiency but also are straightforward to implement.

We use the solution of the deterministic fluid model to develop an effective approximation method for the same system with stochastic production, warmup, and demand processes. We combine the solution of the deterministic fluid model with the optimal base-stock level for a machine with exponential production and demand inter-arrival times to propose closed-form expressions for the approximate control policy. Through a range of numerical experiments, we show that our approximation method provides a straightforward but effective approximation for controlling the systems with stochastic demand inter-arrival, production, and warmup times in an energy-efficient manner. Specifically, the deterministic fluid model accurately determines the optimal policy type of the stochastic counterpart as either Working-Off or Working-Idle in approximately 89.3% of the problem instances evaluated in our numerical analysis. It is further observed that the approximation method estimates the control parameters accurately. The mean absolute percentage error between the costs derived from the approximate policies and the optimal policies is recorded at 1.37% across 729 distinct instances examined. Moreover, the solution time for the proposed approximation is around 250 times faster than that required to determine the optimal policy solving an LP formulation of the associated Markov Decision Process.

In this work, the way we model the deterministic energy-mode control problem is similar to the classical Economic Production Quantity (EPQ) model. We show that the solution of the EPQ model with backlog yields the optimal values of the thresholds of the Working-Off policy by appropriately transforming the system parameters. Connecting the energy mode control problem to the EPQ problem provides an accessible avenue for introducing energy-aware production control concepts and deriving managerial insights. In addition, we show the effects of the system parameters on the control policy, and a Working-Off policy is optimal only when utilization is below a specific threshold provided in the closed form. We also demonstrate that a hybrid policy involving working, off, and idle modes cannot be optimal for the problem studied in this work. This comprehensive understanding may help practitioners avoid sub-optimal strategies and adopt more effective, streamlined approaches.

This study makes contributions to the existing literature in two primary areas. First, we provide the deterministic fluid modeling approach as a new and versatile method for the well-studied energy mode control problem. We expose the connection between the classical EPQ model and the deterministic turning on/off control model and discuss how these two problems are interrelated. Second, we propose an effective approximate policy for a system operating with stochastic production, warmup, and demand processes using the closed-form expressions derived from the deterministic fluid model. The approximate method gives near-optimal results and requires significantly lower computational effort compared to solving a large-scale Markov Decision Process (MDP) or a simulation-based optimization problem.

The organization of the remaining part of this paper is as follows. Section 2 reviews the related literature. Section 3 gives the model and the relevant assumptions. In Section 4, the model is analyzed under the pure and hybrid policies, and the optimal thresholds and the cost are derived in closed form. Section 5 presents the approximation method to control a machine with stochastic production, warmup, and demand inter-event times. The numerical experiments are discussed in Section 6. Finally, Section 7 presents the conclusions.

2. Literature review

Energy efficiency in manufacturing has received significant attention in the literature in recent years, with numerous comprehensive reviews providing insights into relevant studies, such as Gunasekaran and Spalanzani (2012), Gahm et al. (2016), Biel and Glock (2016), Bänsch et al. (2021). This section focuses on two streams within the relevant literature: (i) Energy-aware production control in a stochastic operational setting and (ii) Energy-aware production control in a deterministic operational setting.

2.1. Energy-aware production control in a stochastic operational setting

In this literature stream, studies typically consider stochastic demand inter-arrival times, production times, and warmup times and carry out the necessary analyses through either simulation methods or stochastic analytical models.

As one of the seminal works employing a simulation-based approach to analyze buffer-based policies for energy-saving on/off control of machine tools, Frigerio and Matta (2015) demonstrate that incorporating buffer information into a control policy can achieve significant energy-saving. Renna (2018) explores the impact of energy-efficient production control on the production performance in a pull system, in contrast to a push system as studied in Frigerio and Matta (2015). On the other hand, Frigerio et al. (2020, 2023) extend the offline energy-efficient control policy proposed by Frigerio and Matta (2015) to online and serial line applications.

The relevant literature also includes several studies employing analytical models instead of simulation-based approaches. As a precursor in this research stream, Wang et al. (2016) adopt a Markovian model to assess the efficacy of on/off switching policies in a serial manufacturing system. Jia et al. (2016) concentrate on a subset of a serial line

comprising Bernoulli machines and devise an approximate analytical method to determine an on/off switching policy based on buffer levels for such a machine group. Cui et al. (2021) propose a Markov model to investigate the relationship between production rate and energy consumption in serial production lines. Loffredo et al. (2023a) focus on an energy-aware production control problem for parallel machines and characterize turn-on/off policies depending on the number of parts in the buffer. Tan et al. (2023b) analyze a make-to-stock system with multiple energy modes and exponentially distributed demand inter-arrival, production, and warmup times. The authors model this system as a Markov decision process and, through extensive numerical experiments, demonstrate that the integrated optimal energy and production control policy for such systems is a buffer-based threshold policy. Specifically, their numerical results reveal that the optimal policy is either the Working-Off or Working-Idle policy. Özkan and Tan (2024) establish the optimality of the threshold policy in conditions of heavy traffic for a system operating across working, off, warmup, and idle energy modes. They further demonstrate that either the Working-Off or Working-Idle policy is optimal under these conditions. Nevertheless, the optimal policy, including the determination of its necessary thresholds, must still be computed through numerical methods. Tan et al. (2023a) expand upon the model presented in Tan et al. (2023b) to incorporate phase-type inter-event times using a Markov decision model. The authors demonstrate that the optimal policy is a state-dependent threshold policy, with the buffer-based thresholds contingent on the phase of the demand inter-arrival times. The stochastic version of the model studied in this paper is similar to the ones analyzed in Tan et al. (2023a,b), Özkan and Tan (2024). Our work deviates from these studies in assuming deterministic production, demand, and warmup rates instead of stochastic demand inter-arrival, production, and warmup times. Moreover, we derive the thresholds analytically in closed form rather than using a computational approach.

2.2. Energy-aware production control in a deterministic operational setting

One of the earlier studies on energy-aware production control in a deterministic operational setting is presented in Mouzon et al. (2007). The authors consider a single machine operating in a job shop environment and introduce diverse on/off control policies contingent upon the machine's idle times. Mouzon and Yıldırım (2008) extend their previous effort by addressing a multi-objective optimization problem that involves minimizing total energy consumption and tardiness. Mashaei and Lennartson (2014) address designing an on–off control strategy for operating robots in a cyclic flow shop plant and developing a mixed integer linear programming model with the energy monitor as an objective function.

The previously mentioned studies primarily focus on operational decisions, specifically addressing energy-aware scheduling problems within manufacturing systems. However, in the deterministic literature, several studies have also integrated various environmental considerations into the Economic Order Quantity (EOQ) model. Hovelaque and Bironneau (2015) propose an EOQ model that considers the link between an optimal order quantity, total carbon emissions, price and environmental-dependent demands. Arslan and Turkay (2013) revise the standard EOQ model to incorporate sustainability considerations that include environmental and social criteria such as carbon tax, carbon emission penalties, incentives, etc., in addition to the conventional economic considerations. This line of research is further expanded by the work of Liao and Deng (2018), Liao and Li (2021), incorporating stochastic demand into their analyses, and by Yassine (2020), addressing the implications of stochastic product quality. On the other hand, Condeixa et al. (2022) extend the classical EOQ model application to closed-loop supply chain systems, considering some environmental costs in logistics, inventory phase, manufacturing, and remanufacturing decisions. A comprehensive review of this literature stream is provided by Battini et al. (2014) and Becerra et al. (2021).

Our study also pertains to the classical Economic Production Quantity (EPQ) models. When backorders are allowed, inventory dynamics of the EPQ model introduced by Cárdenas-Barrón (2001) will be similar to that of the deterministic energy-mode control considered in this study. Nevertheless, their objective functions differ from ours since the energy-mode control problem considers energy costs in different modes, and the EPQ model considers a setup cost. By setting the setup cost equal to a term that depends on the energy costs in warmup and off modes and the warmup rate, we show that the EPQ model yields the same optimal level for the upper and lower thresholds of the Working-Off policy as the optimal inventory and backlog levels. The closest study in the EPQ literature to the model we analyzed is the one presented by Nguyen et al. (2023). However, their approach to defining costs and decision variables diverges from ours, and they do not account for an idle and a warmup mode for the machine and a buffer-based, energy-aware control policy. Beyond the scope of their work, our work investigates energy mode control policies and conducts a comprehensive numerical analysis to evaluate the performance of control policies derived from deterministic models in managing systems with stochastic dynamics. Last but not least, Nezami and Heydar (2019) consider a variable unit production cost that includes energy consumption charge, which is a function of production time and time-of-use and alternates between two prices during peak and off-peak hours. They propose a mixed-integer nonlinear mathematical model to minimize the overall cost of the system and determine the optimum production run length.

Considering Table 1, we can restate and summarize our contribution as follows: Unlike simulation approaches or analytical approaches that require a computational method to determine the control parameters, this study contributes to the energy-aware mode control literature by proposing a deterministic fluid model that yields closed-form expressions for the thresholds and the optimal cost. Through a series of numerical experiments, we also reveal that the proposed deterministic model offers a straightforward yet effective approximation for controlling systems with stochastic demand inter-arrival, production, and warmup times. This approach facilitates energy-efficient management for systems with stochastic dynamics while maintaining a high degree of simplicity in its implementation. To the best of our knowledge, our work represents the first attempt to address this specific focus in the literature. The closed-form expressions also enable us to clarify the influence of system parameters on control parameters, thereby enabling practitioners to make informed decisions based on a precise understanding of parameter effects in both deterministic and stochastic operational settings. Hence, this constitutes a notable practical contribution of the study. We also contribute to the deterministic production control literature by showing the connection between the control policies and inventory dynamics in these models. We show the relationship between the EPQ and energy mode control model to obtain the same optimal thresholds.

3. Model

We consider a deterministic fluid model to determine a single machine's optimal buffer-based energy mode control policy. The machine can operate in four different energy modes: Working (W), Off (O), Warmup (R), and Idle (I). The energy consumption costs in off, on, idle, and warmup periods are c_{O}, c_{W}, c_{I}, and c_{R} respectively. The production rate is μ, and the demand rate is λ where $\lambda < \mu$. When the machine is off, there is a warmup delay of $1/\tau$ to start producing again. The inventory/backlog level at time t is $x(t)$, and the inventory holding and backlog costs are c^+ and c^-, respectively. Table 2 gives the main model parameters and variables.

In the long run, production should be equal to demand to prevent the inventory and backlog levels from increasing without a bound. Since $\mu > \lambda$, the machine will operate $u = \lambda/\mu$ fraction of the time where u is referred to as utilization. The remaining time can be used to turn off the machine and then warm up or keep it idle. The energy

Table 1
Summary of reviewed studies.

Study	Model type	Modeling approach	Objective function	Closed-form solution	Decision variables	Energy modes			
						On	Off	Warm-up	Idle
Frigerio and Matta (2015)	S	SIM	Machine's expected energy consumed per part	×	Cycle times to control energy modes	✓	✓	✓	×
Wang et al. (2016)	S	SIM	Energy consumption costs and inventory-related costs	×	Cycle times to control energy modes	✓	✓	✓	✓
Jia et al. (2016)	S	MM	Machine's expected energy consumed per part	×	Buffer levels to control energy modes	✓	✓	✓	×
Renna (2018)	S	SIM	Machine and buffer energy consumption costs	×	Buffer levels to control energy modes	✓	✓	✓	✓
Liao and Deng (2018)	S	EOQ	Environmental effects- and inventory-related costs	×	Order quantity	×	×	×	×
Frigerio et al. (2020)	S	SIM	Energy consumption costs and inventory-related costs	×	Cycle times to control energy modes	✓	✓	✓	✓
Yassine (2020)	S	EOQ	Environmental effects- and inventory-related costs	×	Order quantity	×	×	×	×
Liao and Li (2021)	S	EOQ	Environmental effects- and inventory-related costs	×	Order quantity	×	×	×	×
Cui et al. (2021)	S	MM	Energy consumption costs	×	Buffer levels to control energy modes	✓	✓	×	×
Frigerio et al. (2023)	S	SIM	Energy consumption costs and inventory-related costs	×	Buffer levels to control energy modes	✓	×	×	✓
Loffredo et al. (2023a)	S	MM	Energy consumption costs and inventory-related costs	×	Buffer levels to control energy modes	✓	✓	✓	✓
Tan et al. (2023b)	S	MM	Energy consumption costs and inventory-related costs	×	Buffer levels to control energy modes	✓	✓	✓	✓
Tan et al. (2023a)	S	MM	Energy consumption costs and inventory-related costs	×	Buffer levels to control energy modes	✓	✓	✓	✓
Özkan and Tan (2024)	S	MM	Energy consumption costs and inventory-related costs	×	Buffer levels to control energy modes	✓	✓	✓	✓
Cárdenas-Barrón (2001)	D	EPQ	Inventory-related costs	✓	Production Quantity	×	×	×	×
Mouzon et al. (2007)	D	MP	Energy consumption costs	×	Production Scheduling	✓	✓	×	×
Mouzon and Yıldırım (2008)	D	MP	Energy consumption costs and tardiness	×	Production Scheduling	✓	✓	×	✓
Arslan and Turkay (2013)	D	EOQ	Environmental effects-, social effects-and inventory-related costs	✓	Order quantity	×	×	×	×
Mashaei and Lennartson (2014)	D	MP	Energy consumption costs	×		✓	✓	×	✓
Hovelaque and Bironneau (2015)	D	EOQ	Environmental effects- and inventory-related costs	✓	Order Quantity	×	×	×	×
Nezami and Heydar (2019)	D	MP	Energy consumption costs and inventory-related costs	×	Production scheduling	×	×	×	×
Condeixa et al. (2022)	D	EOQ	Environmental effects-, social effects-and inventory-related costs	✓	Order quantity	×	×	×	×
Nguyen et al. (2023)	D	EPQ	Energy consumption costs and inventory-related costs	×	Cycle times to control energy modes, Production Rate	✓	✓	×	×
This work	D	EPQ	Energy consumption costs and inventory-related costs	✓	Buffer levels to control energy modes	✓	✓	✓	✓

Note: S: Stochastic model, D: Deterministic model, SIM: Simulation approach, MM: Markovian approach, MP: Mathematical programming, EOQ: Economic order quantity approach, EPQ: Economic production quantity approach.

Table 2
Model parameters and notations.

Notation	Definition	Notation	Definition
λ:	Demand rate	μ:	Production rate
u:	Utilization, i.e., λ/μ	τ:	Warmup rate
c_W:	On-period energy cost per time	c_O:	Off-period energy cost per time
c_R:	Warmup-period energy cost per time	c_I:	Idle-period energy cost per time
c^+:	Inventory carrying cost per unit per time	c^-:	Backlog cost per unit per time
t_W:	On time	t_O:	Off time
t_R:	Warmup time	t_I:	Idle time
T:	Total length of on/off/idle cycle	S_{off}:	OFF Threshold
S_{on}:	ON Threshold	S_{idle}:	IDLE Threshold
S:	Backlog level when the backlog level starts increasing	$x(t)$:	Inventory/backlog level at time t
X^+:	Average inventory level	X^-:	Average backlog level
$EC_i(S_{on}, S_{off})$:	Energy cost per cycle under policy $i \in \{wo, wi\}$	$TC_i(S_{on}, S_{off})$:	Total cost per cycle under policy $i \in \{wo, wi\}$

mode control policy determines when to turn it off and back on and when to hold the machine idle and then resume production, depending on the inventory/backlog level.

The energy mode of the machine is controlled according to a threshold policy depending on the inventory/backlog level. We consider two types of control policies: pure policies that use either Off or Idle mode with the Working mode, i.e., Working-Off and Working-Idle policies, and hybrid policies that use Working, Off, and Idle modes simultaneously.

Of these pure policies, the Working-Off policy has two control parameters: S_{off} and S_{on}. If $S_{off} > S_{on}$, when the inventory level reaches S_{off}, the machine is turned off, and the inventory level starts decreasing. The machine is turned on when the inventory level decreases to S_{on}. Since there is a warmup delay, the inventory level starts increasing when the backlog level reaches $S = S_{on} - \lambda/\tau$. When $S_{off} = S_{on}$, the resource is turned off when the inventory level reaches S_{off} and it is immediately turned on and the warmup starts. When

the warmup is completed, it starts producing and continues until the inventory level reaches S_{off}.

Under the Working-Idle policy, the machine is never turned off and operates between the working and idle modes to meet the demand exactly. According to the Working-Idle policy, when the inventory/backlog level reaches the idle threshold S_{idle}, the machine is put in idle mode, and when the inventory/backlog level decreases below S_{idle}, it starts producing. In the fluid case, since $\mu > \lambda$, operating in the Working-Idle policy keeps the inventory/backlog level exactly at S_{idle}. However, when this policy is implemented for discrete manufacturing, the machine is put in idle mode at S_{idle} and starts producing immediately when demand arrives, and the inventory/backlog level decreases to $S_{\text{idle}} - 1$.

Lastly, the energy mode can also be controlled using a hybrid policy that simultaneously incorporates on, off, and idle modes. In such cases, the number of thresholds depends on the specific type of hybrid policy implemented. Let us suppose that the machine is controlled by the Working-Idle-Off with thresholds S_{off}, S_{idle}, and S. Under this hybrid policy, the machine is turned off when the inventory level reaches S_{off}. If the machine is turned off, it is turned on when the inventory decreases to $S_{\text{on}} = S_{\text{idle}} + \lambda/\tau$, and the warmup starts. The machine is put in idle mode when the inventory level decreases to S_{idle}. The machine starts producing whenever the inventory level drops to S. Note that the hybrid Working-Off-Idle policy can operate as the pure Working-Off or Working-Idle policy by selecting its thresholds appropriately. Specifically, if S_{idle} is set above $S_{\text{off}} - \lambda/\tau$, the machine is put into the idle mode when the inventory level reaches S_{off} and starts producing when it decreases to S_{idle} without turning it off.

Since the demand, production, and warmup rates are deterministic, setting the thresholds determines the on, off, and idle times. Accordingly, the buffer-based policies can also be expressed as time-based policies using the on, off, and idle times as the control parameters for the deterministic case. Given the problem structure above, the objective is to determine the optimal energy mode control policy and its parameters to minimize the total energy consumption due to operating in different energy modes, average inventory holding, and average backlog costs per cycle.

4. Energy mode control of a single machine based on the inventory/backlog level

In this section, we analyze the system under the pure and hybrid policies, derive the total cost per cycle for each policy type, and then determine the relevant optimal thresholds. We then show that pure policies are always better than hybrid ones. Consequently, by comparing the solutions of the pure policies, we give our main results for the optimal policy and its parameters to be used depending on the system parameters.

4.1. Analysis of the working-off policy

Fig. 1 depicts how the inventory/backlog level changes over time under the Working-Off policy for given thresholds S_{on} and S_{off} when $S_{\text{on}} > S_{\text{off}}$. Similarly, when $S_{\text{off}} = S_{\text{on}}$, the inventory/backlog dynamics are equivalent to the one depicted in Fig. 1 with $t_{\text{O}} = 0$. The total cost per cycle can be expressed in terms of these thresholds and the system parameters using the inventory/backlog dynamics of the Working-Off policy. The results are given in Lemma 1.

Lemma 1. *The total energy, inventory, and backlog cost per cycle for a machine operating with the Working-Off policy, $TC_{\text{wo}}(S_{\text{on}}, S_{\text{off}})$ is given as*

$$TC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}) = \begin{cases} c_{\text{W}}u + c_{\text{O}}(1-u) + \frac{(c_{\text{R}}-c_{\text{O}})\lambda(1-u)}{\tau(S_{\text{off}}-S_{\text{on}})-\lambda} + c^+X^+ + c^-X^-, & \text{when } S_{\text{off}} > S_{\text{on}}, \\ c_{\text{W}}u + c_{\text{R}}(1-u) + c^+X^+ + c^-X^-, & \text{when } S_{\text{off}} = S_{\text{on}}, \end{cases} \tag{1}$$

where the average inventory and backlog levels are

$$X^+ = \begin{cases} \frac{S_{\text{off}}^2}{2(S_{\text{off}}-S)}, & \text{when } S_{\text{off}} \geq 0, \\ 0 & \text{otherwise}, \end{cases} \quad \text{and} \quad X^- = \begin{cases} \frac{S^2}{2(S_{\text{off}}-S)}, & \text{when } S \leq 0, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $S = S_{\text{on}} - \frac{\lambda}{\tau}$.

The proof is given in Appendix A.

The objective of the energy mode control problem for the Working-Off policy is to minimize the total energy, inventory holding, and backlog cost per cycle given in Eq. (1) by determining the optimal values of the on/off thresholds S_{off} and S_{on} subject to the requirement that the selection of the thresholds should yield non-negative on and off duration. Mathematically, this optimization problem can be written as follows:

$$\min_{S_{\text{off}}, S_{\text{on}}} \quad TC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}), \tag{3a}$$

$$\text{s.t.} \quad S_{\text{off}} \geq S_{\text{on}}. \tag{3b}$$

Optimizing this specified problem with respect to the threshold levels $S_{\text{off}} \geq S_{\text{on}}$ would result in Theorem 1.

Theorem 1. *The optimal thresholds and corresponding optimal cost for a machine operating under the Working-Off policy are determined as follows:*

$$S_{\text{off}}^* = \sqrt{\frac{2\lambda(1-u)}{\tau}\frac{c^-(c_{\text{R}}-c_{\text{O}})}{c^+(c^++c-)}}, \tag{4}$$

$$S_{\text{on}}^* = \frac{\lambda}{\tau} - \sqrt{\frac{2\lambda(1-u)}{\tau}\frac{c^+(c_{\text{R}}-c_{\text{O}})}{c^-(c^++c-)}}, \tag{5}$$

$$S^* = -\sqrt{\frac{2\lambda(1-u)}{\tau}\frac{c^+(c_{\text{R}}-c_{\text{O}})}{c^-(c^++c-)}}, \tag{6}$$

$$TC_{\text{wo}}^* = c_{\text{W}}u + c_{\text{O}}(1-u) + \sqrt{\frac{2(c_{\text{R}}-c_{\text{O}})\lambda(1-u)}{\tau}\frac{c^+c^-}{c^++c^-}}. \tag{7}$$

The proof is given in Appendix B.

The proof of Theorem 1 reveals that $S_{\text{off}}^* > S_{\text{on}}^*$. When $S_{\text{off}} > S_{\text{on}}$, the durations of the on, off, and warmup periods are given by $t_{\text{W}} = \frac{S_{\text{off}}-S_{\text{on}}+\frac{\lambda}{\tau}}{\mu-\lambda}$ and $t_{\text{O}} = \frac{S_{\text{off}}-S_{\text{on}}}{\lambda}$, and $\frac{1}{\tau}$, respectively. These durations are always strictly positive since $\mu > \lambda$, and $\tau > 0$.

In our setting, the inventory holding and backlog costs, c^- and c^+, are used as parameters that reflect how the desired production quantity will be met in the short and long term. In other words, they determine the number of times the machine will be turned on or off to meet the demand in a given period. The next result gives the length of the on–off cycle, T_{wo} when the optimal thresholds given in Theorem 1 are used. The frequency of turning off the machine is $\frac{1}{T_{\text{wo}}}$.

Lemma 2. *The length of the on–off cycle, T_{wo} for a machine operating with the optimal thresholds for the Working-Off policy is given as:*

$$T_{\text{wo}} = \sqrt{\frac{2(c_{\text{R}}-c_{\text{O}})}{\lambda(1-u)\tau}}\sqrt{\frac{1}{c^+}+\frac{1}{c^-}}. \tag{8}$$

Proof. The length of an on–off cycle is the sum of the working, off, and warmup periods, i.e., $T_{\text{wo}} = t_{\text{O}} + t_{\text{O}} + t_{\text{R}} = \frac{S_{\text{off}}-S}{\lambda(1-u)}$. Inserting the optimal thresholds given in Theorem 1 yields the above result. □

Eq. (8) shows that for given energy costs, production, demand arrival, and warmup times, increasing inventory holding and backlog costs decreases the length of the on–off period and, equivalently, increases the frequency of times the machine is turned off. Consequently, the same production target is met in a shorter period, e.g., in every shift, as opposed to meeting the production target in a longer period, e.g., in a given number of shifts or days. The holding and backlog costs

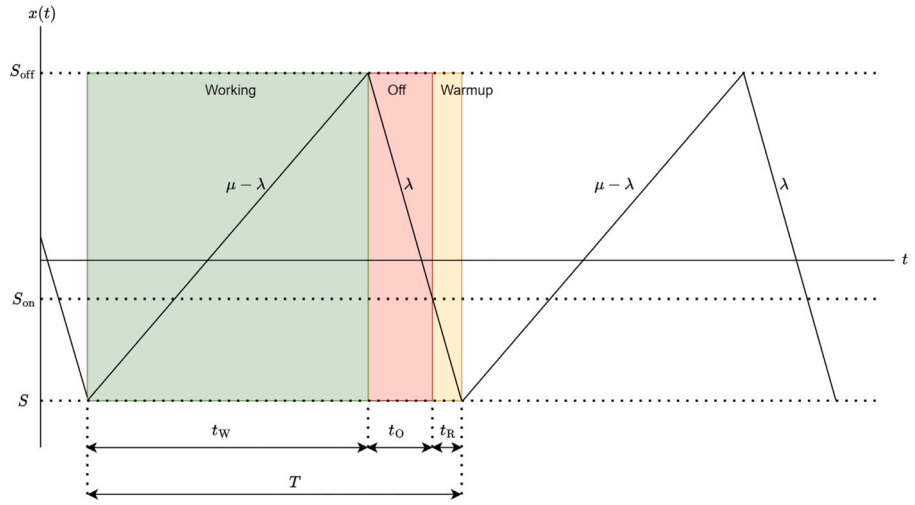

Fig. 1. Inventory/Backlog dynamics under Working-Off Policy when $S_{on} > S_{off}$.

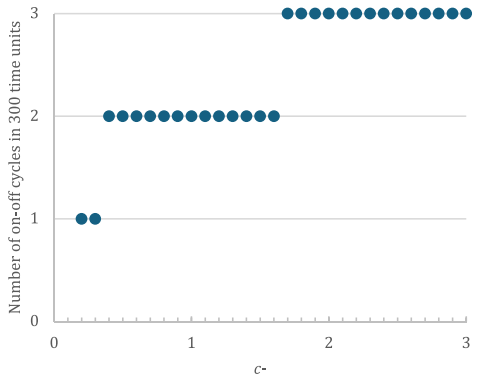

Fig. 2. Effect of the backlog cost on the number of on–off cycles to be completed in a period of 300-time units when $\mu = 1$, $\lambda = 0.6$, $c^+ = 1$, $c_O = 0$, and $c_R = 150$.

are used to tune the optimal policy to balance energy cost minimization and service goals. For instance, Fig. 2 shows the number of on–off cycles to be completed in 300-time units as the backlog cost increases for a specific case.

4.2. Analysis of the working — idle policy

The Working-Idle policy switches the machine between the working and idle energy modes depending on the idle threshold. Therefore, the objective for the energy mode control problem for the Working-Idle policy is choosing S_{idle} to minimize the total energy, backlog, and inventory costs:

$$\min_{S_{idle}} TC_{wi}(S_{idle}) = c_W u + c_I(1-u) + c^+X^+ + c^-X^-. \tag{9}$$

For the deterministic case, since $\mu > \lambda$, the machine operates $u = \frac{\lambda}{\mu}$ fraction of the time and keeps the inventory/backlog level at S_{idle}. Since the objective function includes the inventory and backlog costs, the solution of the deterministic model for the Working-Idle policy yields $S^*_{idle} = 0$. That is, the machine keeps the inventory/backlog level at 0 by producing to meet the demand exactly and keeping the machine idle in the remaining time. As a result, the average inventory and backlog levels are 0. Correspondingly, the total cost under the Working-Idle policy TC^*_{wi} is:

$$TC^*_{wi} = c_W u + c_I(1-u). \tag{10}$$

4.3. Analysis of the hybrid control policies incorporating working, idle, and off modes

To determine the optimal policy, the minimum costs that can be obtained under all possible energy mode control policies must be compared. Our next result shows that controlling the energy mode of a machine with hybrid policies that use working, idle, and off modes simultaneously is never optimal.

Theorem 2. *Employing a Working-Off or Working-Idle policy is always better than hybrid control policies incorporating working, off, and idle modes.*

The proof is given in Appendix C.

In a stochastic framework where the inter-event times associated with warmup, demand, and production processes are governed by exponential distributions, Özkan and Tan (2024) have analytically demonstrated the suboptimality of hybrid policies. Theorem 2 confirms the same result in a deterministic environment using a much simpler approach.

4.4. Optimal energy mode control policy

Since Theorem 2 ensures that the optimal policy is either a Working-Off policy or a Working-Idle policy, the optimal energy mode control policy is determined by comparing the minimum costs that can be obtained by using these two policies. Specifically, if the minimum cost obtained under the Working-Off policy is lower than the minimum cost of the Working-Idle policy, the optimal policy Π^* is controlling the machine with the Working-Off policy and otherwise with the Working-idle policy, i.e.,

$$\Pi^* = \begin{cases} \text{Working-Off}, & \text{if } TC^*_{wo} \le TC^*_{wi}, \\ \text{Working-Idle}, & \text{if } TC^*_{wo} > TC^*_{wi}. \end{cases} \tag{11}$$

Accordingly, the optimal cost TC^* is the minimum of the lowest costs that can be obtained with the Working-Off and Working-Idle policies:

$$TC^* = \min\{TC^*_{wo}, TC^*_{wi}\}, \tag{12}$$

where TC^*_{wo} is given in Eq. (7) and TC^*_{wi} is given in Eq. (10).

4.5. Effect of the system parameters

The system parameters determine which policy to use as the optimal control policy. Consequently, they affect the optimal cost and the thresholds. Our next results show that when the utilization is below a certain level that depends on the system parameters, the Working-Off policy is optimal.

Table 3
Effects of system parameters on the optimal cost, threshold levels, and the length of the on–off period for the Working-Off policy and the optimal cost for the Working-Idle policy. (The terms *increasing/decreasing, up/down* indicate whether the first derivative with respect to a particular parameter is *positive/negative* and the second derivative is *positive/negative*, respectively.)

	Inventory related costs		Energy related costs				Rates		
	c^-	c^+	c_R	c_O	c_I	c_W	λ	μ	τ
TC^*_{wo}	Increasing, Down	Increasing, Down	Increasing, Down	–	–	Linear	If $u < 0.5$, Decreasing, Down	Increasing, Down	Decreasing, Up
S^*_{off}	Increasing, Down	Decreasing, Up	Increasing, Down	Decreasing, Down	–	–	If $u < 0.5$, Increasing, Down; Otherwise, Decreasing, Down	Increasing, Down	Decreasing, Up
S^*_{on}	Increasing, Down	Decreasing, Up	Decreasing, Up	Increasing, Up	–	–	If $u > 0.5$, Increasing, Up	Decreasing, Up	–
S^*	Increasing, Down	Decreasing, Up	Decreasing, Up	Increasing, Up	–	–	If $u < 0.5$, Decreasing, Up; Otherwise, Increasing, Up	Decreasing, Up	Increasing, Down
T_{wo}	Decreasing, Up	Decreasing, Up	Increasing, Down	Decreasing, Down	–	–	Decreasing, Up	Decreasing, Up	Decreasing, Up
TC^*_{wi}	–	–	–	–	Linear	Linear	Linear	Decreasing, Up	–

Lemma 3. *When the utilization is greater than the threshold specified in Eq. (13), the Working-Idle policy gives a lower cost than the Working-Off policy's cost. Therefore, the Working-Idle policy is optimal. Otherwise, the Working-Off policy is optimal.*

$$\Pi^* = \begin{cases} \textit{Working-Off}, & \textit{if } u \le 1 - 2\frac{c_R - c_O}{(c_I - c_O)^2}\frac{\lambda}{\tau}\frac{c^+c^-}{c^+ + c^-}, \\ \textit{Working-Idle}, & \textit{if } u > 1 - 2\frac{c_R - c_O}{(c_I - c_O)^2}\frac{\lambda}{\tau}\frac{c^+c^-}{c^+ + c^-}. \end{cases} \tag{13}$$

Proof. Comparing Eq. (7) with Eq. (10) gives the above result directly. □

Lemma 3 shows that the tradeoff among the inventory, backlog, and energy costs determines whether the benefits of turning off a machine can be justified considering the increased inventory backlog and inventory costs due to the induced warmup delay. The result shows that this benefit can be justified only below a specific utilization level.

Our next result shows how the optimal costs, thresholds, and the length of the on–off period change with the system parameters.

Lemma 4. *The effects of system parameters on the optimal cost, threshold levels, and the length of the on/off period for the Working-Off and the optimal cost of the Working-Idle policy are given in Table 3.*

Proof. Determining the signs of the first and the second derivatives with respect to a particular parameter gives the above results directly. □

Table 3 shows that when the optimal policy is the Working-Off policy, $TC^* = TC^*_{wo}$ is an increasing, down function in c^-, c^+, c_R, and λ, that is, the rate of increase in TC^*_{wo} with respect to c^-, c^+, c_R, and λ is slowing, and it is leveling off at some point. However, when the optimal policy is the Working-Idle policy, since the thresholds, average inventory, and backlog levels are 0, the inventory-related costs do not affect the optimal cost. In this case, the optimal cost $TC^* = TC^*_{wi}$ increases linearly with the arrival rate and decreases with an increasing rate with the production rate. Since the machine is never turned off, the off and warmup energy costs do not affect the optimal cost, and the idle energy cost increases the total cost linearly.

Lemma 3 shows that the optimal policy changes based on a threshold determined by the system parameters. As one of the parameters changes, the overall effect on the optimal cost and the thresholds will be a combination of the results for the Working-Off and Working-Idle policies given in Lemma 4.

5. An approximate policy for a system with stochastic production, warmup and demand inter-arrival times

We propose using the solution of the deterministic fluid model in an approximate policy to control production in a system with stochastic production, warmup, and demand inter-arrival times.

The justification for this approximation is predicated on the relatively minor cost of errors incurred in determining optimal parameters due to stochastic variability compared to other associated costs. Note that the total optimal cost function TC^*_{wo} given in Eq. (7) and TC^*_{wi} given in Eq. (10) include energy cost terms that depend on the utilization, energy cost for the warmup period and the inventory and backlog costs depend on the on and off thresholds. Since the per unit time energy costs are much higher than the inventory and backlog costs and the warmup period is much shorter than the working period, the total cost is dominated by the energy costs that depend on the utilization. Since this term will be the same when the system operates with stochastic production, warmup, and demand inter-arrival times, the solution of the deterministic fluid approach can be a good approximation.

We propose using Eq. (11) to determine the policy type based on the solution of the deterministic fluid model. If the optimal policy is the Working-Off policy, the optimal off and on thresholds are given in Eqs. (4) and (5), respectively.

If the optimal policy is the Working-Idle policy, the deterministic solution suggests using $S^*_{idle} = 0$ as the threshold. To account for stochasticity for the production and demand processes, we note that the dynamics of the system operating under the Working-Idle policy are equivalent to a production system operating with a base-stock policy. In this case, we propose using the optimal threshold of the basestock policy for the exponential production and demand inter-arrival times, given in closed form (Caldentey, 2001). Therefore, for the stochastic case, the optimal threshold level to be used is:

$$S^*_{idle} = \left\lceil \frac{\ln(c^+) - \ln(u(c^+ + c^-))}{\ln(u)} \right\rceil. \tag{14}$$

This equation yields $S^*_{idle} = 0$ when $u \le \frac{c^+}{c^+ + c^-}$ which is the optimal solution for the deterministic model. In other words, when $u \le \frac{c^+}{c^+ + c^-}$, the deterministic fluid model yields the optimal solution for the exponential case. Otherwise, using Eq. (14) given in closed-form gives the control policy parameter accounting for different utilization values and inventory and backlog costs.

The advantage of this approach to the methods reviewed in Table 1 is that the proposed method yields the parameters of the control policy, i.e., the on and off thresholds for the Working-Off policy and the idle

threshold for the Working-Idle policy in closed form. The other methods require a large-scale simulation optimization or the solution of a large-size Markov Decision Process problem using value iteration, policy iteration, or linear programming. Therefore, setting up these methods and finding the solution requires considerably more effort and longer computation time. The closed-form expressions for the method given in this study yield the control parameters directly.

In summary, the proposed approximate policy to control the energy mode of a system with stochastic production, warmup, and demand processes uses Eq. (12) to determine the policy type based on the solution of the deterministic fluid model. If the optimal policy is the Working-Off policy, the optimal off and on thresholds are given in Eqs. (4) and (5), respectively. If the optimal policy is the Working-Idle policy, the optimal threshold is given in Eq. (14).

Through a range of numerical instances, we numerically evaluate the performance of this approximation policy in Section 6.

6. Numerical experiments

In this section, we first introduce a case study that motivates our research. Based on this case study, we discuss how the model parameters are established. Utilizing these parameters, we conduct a numerical analysis to evaluate the impact of various system parameters on the optimal cost, policy, and thresholds. Moreover, we conduct an extensive numerical experiment to compare the outcomes of the deterministic model with those of its stochastic counterpart, wherein demand inter-arrival times are governed by an Erlang distribution and production and warmup times are exponentially distributed. The optimal policies for the stochastic variants of our problem are derived using the MDP approach proposed in Tan et al. (2023a,b).

6.1. A case study and parameters for the numerical experiments

This study is motivated by the problem of determining the on–off schedule of the paint oven of the largest plant of a leading automotive manufacturer. The paint shop accounts for 44% of the total energy consumption at the plant. Due to the high energy cost that is 12% of a vehicle's total production cost and environmental concerns, improving energy efficiency is one of the top priorities for the manufacturer. Arriving vehicles at the paint shop are dispatched to either a regular oven or a special vehicle oven, depending on whether the requested color is regular or special. Due to high production requirements for regular vehicles (close to 1000 cars/day), the regular oven must always operate. However, the arrival rate of the vehicles that require special colors is lower compared to the capacity of the special vehicle oven. This lower utilization of the special vehicles oven allows turning the special vehicle oven on and off, depending on the system's state. However, the production requirement for the special vehicles should also be met daily (close to 100 cars/day). In this context, it is imperative to determine the on–off schedule of the special vehicle oven to minimize energy costs while adhering to production requirements.

Considering the special car production requirements in the car manufacturer's painting oven scheduling problem, the demand rate, which corresponds to the desired production rate of the special vehicle line, is $\lambda = 5$ cars/h. It takes 15 min to paint two cars in the special vehicle oven, so $\mu = 8$ cars/h. Correspondingly, the utilization is 62.5%. That allows turning off or keeping the oven idle by 37.5% of the time. When the oven is turned off, the time required to reach the optimal operating temperature necessary for effective drying, referred to as the warmup time, is 2 h. Therefore, the schedule of the on/off periods must be determined not to affect the overall production. Note that in the base scenario used to study the effects of system parameters on the optimal cost, policy, and thresholds, we rescale and normalize these rates relative to the production rate, setting the production rate to 1 and arranging the others accordingly. The main reason behind this step is that it allows for a more straightforward interpretation of the results.

Table 4
The set of parameters considered in the numerical experiments.

Varying parameters	$\lambda = \{0.5, 0.7, 0.9\}$, $\tau = \{0.2, 0.3, 0.4\}$, $c_R = \{100, 200, 300\}$ $c^- = \{0.5, 1, 3\}$, $c_I = \{50, 70, 90\}$, and $cv_d^2 = \{0.1, 0.5, 1\}$
Non-Varying Parameters	$\mu = 1$, $c^+ = 1$, $c_W = 100$, $c_O = 0$, $cv_r^2 = 1$, and $cv_p^2 = 1$

Due to confidentiality considerations, the inventory and energy-related costs are not reported directly in our study. Instead, these costs have been established to accurately reflect their practical relationships in the base scenario, with values set as $c_W = 100$, $c_O = 0$, $c_I = 50$, and $c_R = 150$. As explained in Lemma 2, the inventory holding and backlog costs, c^- and c^+, are used as parameters that reflect whether the desired production quantity will be met in the short term, e.g., producing 20 cars in every shift of 4 h or the long term, e.g., producing 80 cars in 4 shifts. They are set as $c^+ = 1$, $c^- = 3$ for the base case in the numerical experiments.

For the numerical experiments that consider the stochastic variant of this problem, we draw upon similar existing studies in the literature. Consistent with these studies, we model demand inter-arrival, production, and warmup times using phase-type distributions. The Erlang distribution is characterized by two parameters: the shape parameter and the rate parameter. In the context of demand inter-arrival times, we examine three distinct values for the squares of the coefficient of variation cv_d^2, 0.1, 0.5, and 1. Since $cv_d^2 = 1/K_d$ for an Erlang distribution with a shape parameter K_d, we consider Erlang distributions with $K_d = \{10, 2, 1\}$ in the numerical experiments. We model the warmup and production times as random variables with an Exponential distribution with the squared of the coefficient of variations of the warmup and production times equal to 1, i.e., $cv_r^2 = 1$ and $cv_p^2 = 1$, respectively.

Building on the given base scenario, we vary other select parameters as described in Table 4 and perform a comprehensive numerical analysis to investigate the effects of model parameters on overall system performance, as well as the performance of the approximate policies derived from the deterministic policy in the stochastic version. The parameters that remain constant across all numerical instances are listed as the non-varying parameters in Table 4.

6.2. Effect of the system parameters on the deterministic fluid model

In this section, we examine the effects of system parameters for the deterministic fluid model. The effects of these parameters on the optimal cost and the optimal thresholds are presented in Table 2. However, this analysis is conducted to visualize these effects more clearly and to provide a more comprehensive understanding of their implications on the system's performance.

We first investigate the impact of utilization on the optimal cost, optimal policy, and optimal thresholds under varying idle mode energy cost c_I and warmup rate τ. To conduct this sensitivity analysis, we generate two distinct scenarios based on the parameter sets outlined in Table 4. Using the first scenario, we evaluate the system's behavior under two different warmup rates, specifically $\tau = 0.2$ and $\tau = 0.4$, whereas using the second scenario, we assess the system's behavior under two different idle energy mode costs, specifically $c_I = 50$ and $c_I = 70$. In both scenarios, for each warmup rate and warmup cost, we generate 10 distinct problem instances by considering increasing utilization values from 0.5 to 0.9 in increments of 0.05 and 0.99. This results in a total of 40 problem instances examined in this analysis. Note that in these problem instances, adhering to the given base scenario, the remaining parameters are set as follows: $c^+ = 1$, $c^- = 3$, $c_W = 100$, $c_O = 0$, and $c_R = 150$.

Figs. 3 and 4 present the impact of utilization on the optimal cost, optimal policy, and optimal thresholds for various values of the idle energy cost c_I and the warmup rate τ, respectively. These results confirm the analytical findings reported in Lemmas 3 and 4. As given

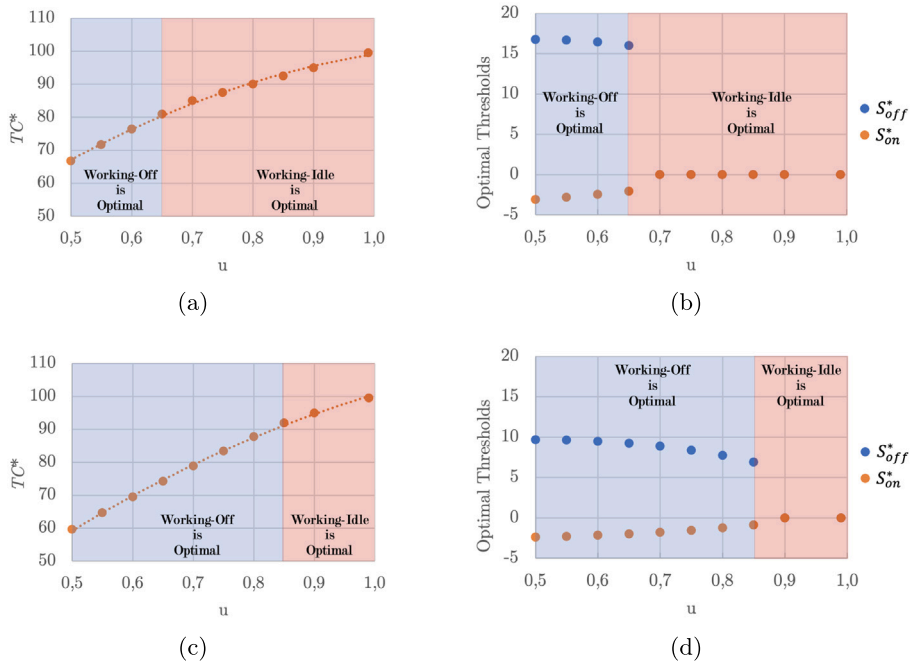

Fig. 3. Effect of utilization on the optimal cost and optimal policy when 3(a) $c_I = 50$ and τ=0.2 and 3(c) $c_I = 50$ and τ=0.4, and effect of utilization on the thresholds when 3(b) $c_I = 50$ and τ=0.2 and 3(d) $c_I = 50$ and τ=0.4. The other parameters are set as: μ=1, λ={0.5, 0.55, … , 0.9, 0.99}, $c^+ = 1$, $c^- = 3$, $c_W = 100$, $c_O = 0$, and $c_R = 150$.

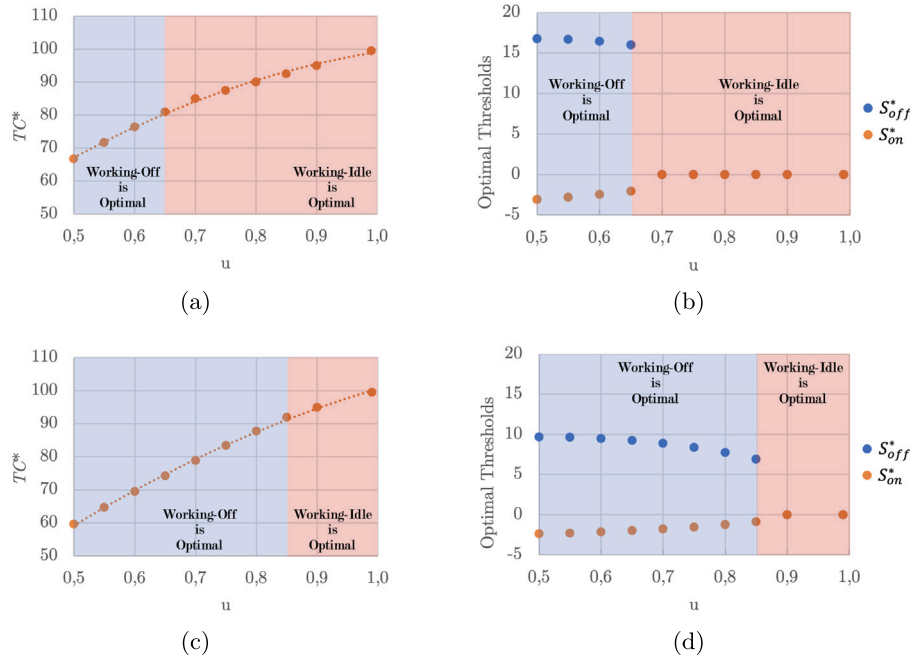

Fig. 4. Effect of utilization on the optimal cost and optimal policy when 3(a) $c_I = 50$ and τ=0.2 and 3(c) $c_I = 70$ and τ=0.2, and effect of utilization on the thresholds when 3(b) $c_I = 50$ and τ=0.2 and 3(d) $c_I = 70$ and τ=0.2. The other parameters are set as: μ=1, λ={0.5, 0.55, … , 0.9, 0.99}, $c^+ = 1$, $c^- = 3$, $c_W = 100$, $c_O = 0$, and $c_R = 150$.

in Lemma 3, the optimal policy switches from the Working-Off policy to the Working-Idle policy when the utilization is above a certain threshold. Fig. 3 shows that this threshold increases with the warmup rate. When the warmup time is shorter, i.e., the warmup rate is higher, the Working-Off policy gives a lower cost for higher utilization values.

On the other hand, Fig. 4 demonstrates that this threshold also increases with the idle energy cost, as given in Lemma 4. When the idle energy cost is higher, the Working-Off policy provides a lower cost for higher utilization values. The thresholds for the Working-Idle policy are 0, following the deterministic production and demand rates. When two thresholds are used, the difference between these thresholds decreases as the utilization increases.

6.3. Performance of the approximate policy for a system with random inter-event times

In this section, we analyze the performance of the approximate policy where the deterministic model determines the policy type, and it is implemented using the closed-form thresholds in a system that has stochastic demand inter-arrival, production, and warmup times through

Table 5
Comparison of ϵ values and accuracy of estimated policies for different cv_d^2 values.

	$cv_d^2 = 0.1$	$cv_d^2 = 0.5$	$cv_d^2 = 1$	All
% of Correctly Estimated Policy	91.0%	88.0%	86.4%	89.3%
Average ϵ	1.98%	1.42%	0.71%	1.37%
Range ϵ	[0.1%, 15.7%]	[0.0%, 13.1%]	[0.0%, 9.9%]	[0.0%, 15.7%]

a comprehensive numerical study. To perform this analysis, we utilize a full factorial experimental design based on the parameters presented in Table 4, leading to 729 distinct problem instances. Through these problem instances, we specifically study the effects of demand variation, demand rate, warmup energy cost, backlog cost, warmup rate, and idle energy cost on the performance of the approximate policy.

Using the problem instances described above, we compare the results of our approximate policy with the optimal policy for the stochastic version, where the optimal solution is obtained by solving a linear program (LP) associated with the Markovian Decision Process modeling of the control problem as given in Tan et al. (2023a,b). The largest LP model solved in the numerical experiments is the one for the case with $cv_d^2 = 0.1$, and it has around 480,000 variables and 521,000 constraints. It takes around 12 s to solve this problem using the CPLEX solver in GAMS on a notebook computer with a 2 GHz Intel-i5 processor and 16 GB RAM. The approximate solution of the same case takes around 0.05 s using MATLAB on the same computer. For all the cases analyzed, the computation time of the approximation method is on average 250 times faster than solving the LP model associated with MDP.

In the numerical analysis, for each problem instance, the relative percentage difference, ϵ, between the optimal cost obtained via the approximation, TC_a^*, and the optimal cost obtained via the MDP approach, TC^*, is calculated, i.e., $\epsilon = 100 \times \frac{|TC_a^* - TC^*|}{TC^*}$. The summary statistics, including the percentage of correctly determined policies, average, and range for these relative cost differences, are presented in Tables 5–7.

Table 5 shows that for the 729 different problem instances considered in this study, the mean absolute deviation between the optimal total cost obtained using the Markov Decision Process (MDP) approach and the total cost obtained using the approximate policy is 1.37%. This deviation ranges from 0% to 15.7%, depending on the specific parameter settings. Furthermore, as shown in the same table, the deterministic model accurately predicts the optimal policy type in approximately 89% of the instances considered in this study. Table 6 shows that the inaccuracies in policy estimations essentially arise from the misclassification of policies in 84 problem instances, accounting for 11.5% of all instances, where Working-Off policies are incorrectly estimated as Working-Idle policies. The table further demonstrates that even in cases where the corresponding optimal policies are incorrectly determined, the average deviation does not exceed 3%.

Table 5 indicates that a decrease in demand arrival uncertainty, which corresponds to an increase in the order of the Erlang distribution, leads to a modest improvement in the accuracy of policy predictions. Specifically, the prediction accuracy increases from 89.30% for demand following an Erlang(2) distribution ($cv_d^2 = 0.5$) to 90.54% for demand following an Erlang(10) distribution ($cv_d^2 = 0.1$). In Table 5, it can also be observed that the average deviation is 2% for $cv_d^2 = 0.1$, 1.4% for $cv_d^2 = 0.5$, and 0.7% for $cv_d^2 = 1$. This implies that the average deviation exhibits a slight increase as demand arrival uncertainty reduces, namely as the order of the Erlang distribution increases. This can be attributed to the fact that the approximate policy establishes thresholds based on the assumption of exponential demand. As the order of the Erlang distribution diverges from one, the accuracy of these threshold estimations decreases, resulting in a corresponding increase in the average deviation.

Table 7 demonstrates that the mean absolute deviation between the optimal total cost derived from the optimal policy for the stochastic demand model and the cost obtained using the approximate policy increases as at least one of the following parameters increases: backlog cost, idle mode energy cost, warmup rate, or demand rate. The table also reveals that this deviation decreases as the warmup mode energy cost increases. In all cases, the mean absolute deviation does not exceed 16%. Notably, in only 6 out of 729 problem instances, which corresponds to only 0.82% of the total instances, this deviation exceeds 10%. When c^-, τ, and λ reach their maximum values, the deterministic model favors the Working-Idle policy to reduce the risk of failing to meet demand and minimize the associated costs. Yet, even when these parameters are set to their maximum levels, the optimal policies obtained through the stochastic models are predominantly Working-Off. Thus, this incompatibility between the optimal and estimated policies results in an increase in the deviation.

Given all these results, one can conclude that the approximate method yields good performance for different varying of stochasticity in the system with minimum computational effort as opposed to determining the optimal policy by solving a large-size Markov Decision Process that requires significant computational effort.

7. Conclusions

This study considers the energy-mode control problem in a deterministic setting. The deterministic fluid model enables us to formulate the problem of minimizing the energy cost due to operating in working, idle, off, and warmup modes with the inventory holding and backlog costs by deciding on the buffer-based thresholds that determine when to turn off and on the machine. This model yields closed-form solutions for the optimal thresholds and the optimal cost. Using the deterministic model, we prove that using pure policies, Working-Off or Working-Idle, is always better than hybrid policies that simultaneously use Working, Off, and Idle modes. We also determine the utilization region where the Working-Off policy is better than the Working-Idle policy regarding cost. Therefore, the inventory dynamics can be described completely by the system parameters and the thresholds for all possible combinations of buffer-based policies.

We use the deterministic model to develop an efficient approximate policy for systems with stochastic inter-event times. Namely, we determine the policy type by using the solution of the deterministic model. If the resulting policy is the Working-Off policy, we give the thresholds in closed form. If the resulting policy is the Working-Idle policy, we use the closed-form solution of the stochastic base-stock model for the exponential case. Our numerical experiments comparing the solution obtained using the approximate policy with the solution of the same problem with stochastic inter-arrival, production, and warmup times show that the deterministic model can be used as an accurate approximation for the stochastic model. The optimal policy type derived from the deterministic model demonstrates an approximate accuracy of 89.3%. Furthermore, our approximation policy requires minimum computational effort since all the parameters are determined in closed form while solving the stochastic problem requires either solving a large-scale MDP or simulation optimization and gives near-optimal results. The solution of the large-scale MDP takes 250 times longer compared to our approach.

In summary, all these comparative analyses demonstrate that the approximate policy built using the deterministic fluid model provides both efficient and effective approximations for its stochastic variants. By leveraging the closed-form expressions, the approximate policies that are efficient, effective, and straightforward to implement for the stochastic versions of the problem can be generated. The ease of use and efficiency of these approximate policies would be instrumental in optimizing operational efficiency in dynamic environments, including but not limited to automotive assembly lines, CNC machine operations, and data storage centers. Furthermore, they would enable practitioners

Table 6
Summary statistics of ϵ for both the estimated and optimal policies.

		Estimated Policy Type Obtained via The Deterministic Model			
		Working-Idle		Working-Off	
Optimal Policy Type Obtained via The Stochastic Model	Working-Idle	Average ϵ	2.1%	Average ϵ	–
		Range ϵ	[0.00%, 5.9%]	Range ϵ	–
		# of Correctly Estimated Policy	78 (10.7%)	# of Correctly Estimated Policy	–
	Working-Off	Average ϵ	3.0%	Average ϵ	1.0%
		Range ϵ	[0.00%, 8.3%]	Range ϵ	[0.00%, 15.7%]
		# of Correctly Estimated Policy	84 (11.5%)	# of Correctly Estimated Policy	567 (77.8%)

Table 7
Summary statistics of ϵ for different levels of c^-, c_I, c_R, τ, and λ.

Parameters		$cv_d^2 = 0.1$		$cv_d^2 = 0.5$		$cv_d^2 = 1$		All	
		Av. ϵ	Range ϵ	Av. ϵ	Range ϵ	Av. ϵ	Range ϵ	Av. ϵ	Range ϵ
c^-	0.5	1.8%	[0.1%, 2.8%]	0.7%	[0.0%, 2.2%]	0.4%	[0.0%, 1.5%]	1.0%	[0.0%, 2.8%]
	1.0	2.0%	[0.1%, 5.4%]	1.1%	[0.1%, 4.3%]	0.6%	[0.0%, 2.8%]	1.2%	[0.0%, 5.4%]
	3.0	2.2%	[0.3%, 15.7%]	2.5%	[0.2%, 13.1%]	1.2%	[0.0%, 9.9%]	2.0%	[0.0%, 15.7%]
c_I	50	1.7%	[0.1%, 7.5%]	1.1%	[0.0%, 5.1%]	0.3%	[0.0%, 2.8%]	1.0%	[0.0%, 7.5%]
	70	2.0%	[0.1%, 15.7%]	1.5%	[0.0%, 13.1%]	0.8%	[0.0%, 9.8%]	1.4%	[0.0%, 15.7%]
	90	2.3%	[0.1%, 15.7%]	1.8%	[0.0%, 13.1%]	1.1%	[0.0%, 9.9%]	1.7%	[0.0%, 15.7%]
c_R	100	2.5%	[0.1%, 15.7%]	1.9%	[0.1%, 13.1%]	1.2%	[0.0%, 9.9%]	1.9%	[0.0%, 15.7%]
	200	1.8%	[0.1%, 13.7%]	1.3%	[0.0%, 11.3%]	0.6%	[0.0%, 8.3%]	1.2%	[0.0%, 13.7%]
	300	1.6%	[0.1%, 8.2%]	1.1%	[0.0%, 6.1%]	0.4%	[0.0%, 3.3%]	1.0%	[0.0%, 8.2%]
τ	0.2	1.8%	[0.1%, 14.9%]	1.2%	[0.0%, 12.7%]	0.5%	[0.0%, 9.9%]	1.2%	[0.0%, 14.9%]
	0.3	2.0%	[0.1%, 14.8%]	1.5%	[0.0%, 12.3%]	0.7%	[0.0%, 9.2%]	1.4%	[0.0%, 14.8%]
	0.4	2.2%	[0.1%, 15.7%]	1.7%	[0.1%, 13.1%]	0.9%	[0.0%, 9.8%]	1.6%	[0.0%, 15.7%]
λ	0.5	0.2%	[0.1%, 0.6%]	0.1%	[0.0%, 0.4%]	0.0%	[0.0%, 0.1%]	0.1%	[0.0%, 0.6%]
	0.7	0.8%	[0.2%, 3.6%]	0.6%	[0.2%, 3.3%]	0.3%	[0.0%, 2.8%]	0.6%	[0.0%, 3.6%]
	0.9	4.9%	[1.8%, 15.7%]	3.5%	[1.0%, 13.1%]	1.8%	[0.0%, 9.9%]	3.4%	[0.0%, 15.7%]
	All	1.98%	[0.1%, 15.7%]	1.42%	[0.0%, 13.1%]	0.71%	[0.0%, 9.9%]	1.37%	[0.0%, 15.7%]

in these fields to make informed decisions based on a better understanding of system dynamics in both deterministic and stochastic operational settings.

The results for the optimal thresholds are related to the Economic Production Quantity with backlog results given in Cárdenas-Barrón (2001). In their work, the authors give the optimal backlog level as $B^* = \sqrt{\frac{2KDh\rho}{b(h+b)}}$. Using our notation, setting $D = \lambda$, $b = c^-$, $h = c^+$, $\rho = \frac{D}{P} = \frac{\lambda}{\mu} = 1-u$, $K = \frac{(c_R - c_O)}{\tau}$ gives the optimal threshold to turn on the machine, S_{on}^* given in Eq. (5). Although a setup cost is not considered in our model, setting a setup cost for each warmup period that is equal to the additional energy cost above the energy cost in the off mode per warmup time gives the optimal backlog level that is equivalent to the optimal lower threshold in the energy mode control problem. The relationship between the energy mode control and the EPQ models can be analyzed further.

An interesting extension of the deterministic approach is to analyze a production system where multiple machines are controlled simultaneously. For a general system, where more than two machines are controlled among many machines, an analytical solution is not available in the literature. Using an MDP approach to obtain a numerical result is impossible due to the rapid increase in the problem size. Using a simulation optimization approach is also computationally demanding. Alternatively, a deterministic fluid simulation that implements the buffer-based control policy can be employed to determine the thresholds. When analyzing the system with stochastic inter-event times, the deterministic solution can be utilized as the starting solution for a simulation optimization approach. It is expected that using the deterministic solution to find the solution for the stochastic system would improve the computational efficiency. However, this extension is left for future research.

In summary, the simple deterministic energy mode control model shows how the energy mode control works and allows us to examine how the system parameters affect the control parameters. Furthermore, the approximation method built using the closed-form solution of the deterministic model yields near-optimal results with significant computational efficiency for a system with stochastic inter-event times.

CRediT authorship contribution statement

Barış Tan: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Oktay Karabağ:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Data availability

No data was used for the research described in the article.

Acknowledgment

This work was supported by TUBITAK (grant number 221M393).

Appendix A. The total cost per cycle for the working off policy

Proof of Lemma 1. Case-1: When $S_{off} > S_{on}$, according to Fig. 1, the lengths of the on and off periods can be expressed in terms of the thresholds and system parameters as follows: $t_W = \frac{S_{off}-S}{\mu-\lambda}$, $t_O = \frac{S_{off}-S}{\lambda} - t_R$ where $S = S_{on} - \lambda t_R$. So, the length of one on/off cycle can be expressed as:

$$T = t_W + t_O + t_R = \frac{(S_{off} - S)}{\lambda(1-u)}. \tag{A.1}$$

The average inventory level during one cycle is calculated by dividing the area above zero, represented by $\frac{S_{\text{off}}^2}{2}\left(\frac{1}{\mu-\lambda}+\frac{1}{\lambda}\right)$, by the cycle length T. Accordingly, when $S_{\text{off}} > 0$, we have:

$$X^+ = \frac{S_{\text{off}}^2}{2\lambda(1-u)}\frac{\lambda(1-u)}{(S_{\text{off}}-S)} = \frac{S_{\text{off}}^2}{2(S_{\text{off}}-S)}. \tag{A.2}$$

Otherwise, $X^+ = 0$.

Analogously, the average backlog level is the area below 0 in one cycle divided by the cycle length. Thus, when $S < 0$, we have:

$$X^- = \frac{S^2}{2\lambda(1-u)}\frac{\lambda(1-u)}{(S_{\text{off}}-S)} = \frac{S^2}{2(S_{\text{off}}-S)}. \tag{A.3}$$

Otherwise $X^- = 0$.

Additionally, the per-cycle energy cost is defined as:

$$EC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}) = \frac{c_{\text{W}}t_{\text{W}} + c_{\text{O}}t_{\text{O}} + c_{\text{R}}t_{\text{R}}}{T}. \tag{A.4}$$

Using $u = \frac{t_{\text{W}}}{T}$ and $1-u = \frac{t_{\text{O}}+t_{\text{R}}}{T}$, the per-cycle energy cost can be expressed in terms of the thresholds and system parameters as follows:

$$EC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}) = c_{\text{W}}u + c_{\text{O}}(1-u) + \frac{(c_{\text{R}}-c_{\text{O}})\lambda(1-u)}{\tau(S_{\text{off}}-S_{\text{on}})-\lambda}. \tag{A.5}$$

Finally, the total cost per cycle is

$$TC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}) = EC(S_{\text{on}}, S_{\text{off}}) + c^+X^+ + c^-X^-. \tag{A.6}$$

Case-2: When $S_{\text{off}} = S_{\text{on}}$, the resource is turned off when the inventory level reaches S_{off} and it is immediately turned on and the warmup starts. When the warmup is completed, it starts producing and continues until the inventory level reaches S_{off}. This dynamic is equivalent to the one depicted in Fig. 1 with $t_{\text{O}} = 0$. Therefore,

$$EC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}) = c_{\text{W}}u + c_{\text{R}}(1-u), \tag{A.7}$$

and

$$TC_{\text{wo}}(S_{\text{on}}, S_{\text{off}}) = EC(S_{\text{on}}, S_{\text{off}}) + c^+X^+ + c^-X^-, \tag{A.8}$$

where X^+ and X^- are

$$X^+ = \begin{cases} \frac{S_{\text{off}}^2\tau}{2\lambda}, & \text{if } S_{\text{off}} \geq 0, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad X^- = \begin{cases} \frac{(S_{\text{off}}-\frac{\lambda}{\tau})^2\tau}{2\lambda}, & \text{if } S_{\text{off}} \leq \frac{\lambda}{\tau}, \\ 0, & \text{otherwise.} \end{cases} \tag{A.9}$$

□

Appendix B. The optimal solution of the optimization problem

Proof of Theorem 1. The two terms of the objective function for the cases $S_{\text{off}} > S_{\text{on}}$ and $S_{\text{off}} = S_{\text{on}}$ given in Eq. (1) are convex, and the constraint given in Eq. (3b) is linear. So, a unique minimum of the problem exists for both cases.

For the case where $S_{\text{off}} > S_{\text{on}}$, the optimal thresholds and corresponding optimal cost for a machine operating under the Working-Off policy are determined as follows:

$$S_{\text{off}}^* = \sqrt{\frac{2\lambda(1-u)}{\tau}\frac{c^-(c_{\text{R}}-c_{\text{O}})}{c^+(c^++c-)}}, \tag{B.1}$$

$$S_{\text{on}}^* = \frac{\lambda}{\tau} - \sqrt{\frac{2\lambda(1-u)}{\tau}\frac{c^+(c_{\text{R}}-c_{\text{O}})}{c^-(c^++c-)}}, \tag{B.2}$$

$$S^* = -\sqrt{\frac{2\lambda(1-u)}{\tau}\frac{c^+(c_{\text{R}}-c_{\text{O}})}{c^-(c^++c-)}}, \tag{B.3}$$

$$TC_{\text{wo},1}^* = c_{\text{W}}u + c_{\text{O}}(1-u) + \sqrt{\frac{2(c_{\text{R}}-c_{\text{O}})\lambda(1-u)}{\tau}\frac{c^+c^-}{c^++c^-}}. \tag{B.4}$$

On the other hand, for the case where $S_{\text{off}} = S_{\text{on}}$, the optimal thresholds and costs are determined as follows:

$$S_{\text{off}}^* = S_{\text{on}}^* = \frac{c^+\lambda}{(c^++c^-)\tau}, \tag{B.5}$$

$$S^* = -\frac{c^-\lambda}{(c^++c^-)\tau}, \tag{B.6}$$

$$TC_{\text{wo},2}^* = c_{\text{W}}u + c_{\text{R}}(1-u) + \frac{c^-c^+\lambda}{2\tau(c^++c^-)}. \tag{B.7}$$

If $TC_{\text{wo},1}^* < TC_{\text{wo},2}^*$, the optimal solution for the Working-Off policy is the optimal solution for the case $S_{\text{off}} > S_{\text{on}}$. Similarly, if $TC_{\text{wo},1}^* > TC_{\text{wo},2}^*$, the optimal solution for the case $S_{\text{off}} = S_{\text{on}}$ yields the solution for the Working-Off policy. If $TC_{\text{wo},1}^* = TC_{\text{wo},2}^*$, the solution is the same as the ones obtained for both cases. Considering all these cases, comparing the two objective functions will reveal which solution is optimal under specific conditions. Hence, we proceed with a comparative analysis of these two objective functions and get the following:

$$TC_{\text{wo},2}^* - TC_{\text{wo},1}^* = A + B - \sqrt{4AB}, \tag{B.8}$$

where $A = (1-u)(c_{\text{R}}-c_{\text{O}})$ and $B = \frac{c^-c^+\lambda}{2\tau(c^++c^-)}$. It should be noted that A and B are always positive due to the assumptions employed in the model for the parameters. As $A+B-\sqrt{4AB} \geq 0$, $\forall A, B \in \mathbb{R}^+$, it follows that the optimal cost for the case $S_{\text{off}} = S_{\text{on}}$ given in Eq. (B.7) is always greater than or equal to the optimal cost for the case $S_{\text{off}} > S_{\text{on}}$ given in Eq. (B.4). Consequently, under the Working-Off policy, the optimal thresholds and corresponding optimal cost are determined using the optimal solution for the case $S_{\text{off}} > S_{\text{on}}$ given in Eqs. (B.1)–(B.4). □

Appendix C. Suboptimality of hybrid policies

In a deterministic setting, the hybrid policies Working-Idle-Off, Working-Idle-Off-Idle, and Working-Off-Idle are essentially equivalent. When all model parameters are held constant, the sequence in which the energy modes are employed — whether it is Working-Idle-Off or Working-Off-Idle — does not influence the duration spent in each energy mode. Furthermore, the Working-Idle-Off-Idle policy can be interpreted as a subdivision of the idle time within the Working-Off-Idle framework into multiple segments. These types of policies, and similar ones, can be further developed. Since the time allocated to each energy mode remains unchanged across these policies when the parameters are unchanged, the total cost per cycle remains consistent, leaving the optimal cost unaffected. In such a case, the only variation will be in the order of the threshold levels. As a result, the selection of energy modes for system management is the crucial aspect, rather than the specific sequence in which these modes are applied. This phenomenon directs our attention exclusively towards the Working-Off-Idle policy, ensuring compatibility with the existing literature. As a result, in the subsequent paragraphs, we focus solely on the Working-Off-Idle policy and provide a detailed analysis based on this policy.

Fig. C.5 depicts how the inventory/backlog level changes over time under the Working-Off-Idle policy. Under the Working-Idle-Off policy, the machine is controlled with three thresholds, S_{off}, S_{idle}, and S. Depending on these thresholds, a machine operates in working, off, warmup and idle modes or only in working and idle modes.

The machine is turned off or idle when the inventory level reaches S_{off}. If the machine is turned off, it is turned on when the inventory level is $S_{\text{on}} = S_{\text{idle}} + \frac{\lambda}{\tau}$ and the warmup starts. The machine starts producing when the inventory level decreases to S. If $S_{\text{idle}} + \frac{\lambda}{\tau} > S_{\text{off}}$, the implied warmup time $t_{\text{R}} = T - t_{\text{W}} - t_{\text{O}} - t_{\text{I}}$ would be less than the given warmup time $\frac{1}{\tau}$. Since this is not possible, if $S_{\text{idle}} + \frac{\lambda}{\tau} > S_{\text{off}}$, the machine operates only in Working and Idle modes. Otherwise, it can operate in Working, Off, and Idle modes. Thus, if $S = S_{\text{idle}}$, this policy will be equivalent to the Working-Off policy. Similarly, if $S_{\text{off}} = S_{\text{idle}}$, this policy will be equivalent to the Working-Idle policy.

Fig. C.5 shows that the lengths of the on, off, idle periods can be expressed in terms of the thresholds and system parameters as $t_{\text{W}} =$

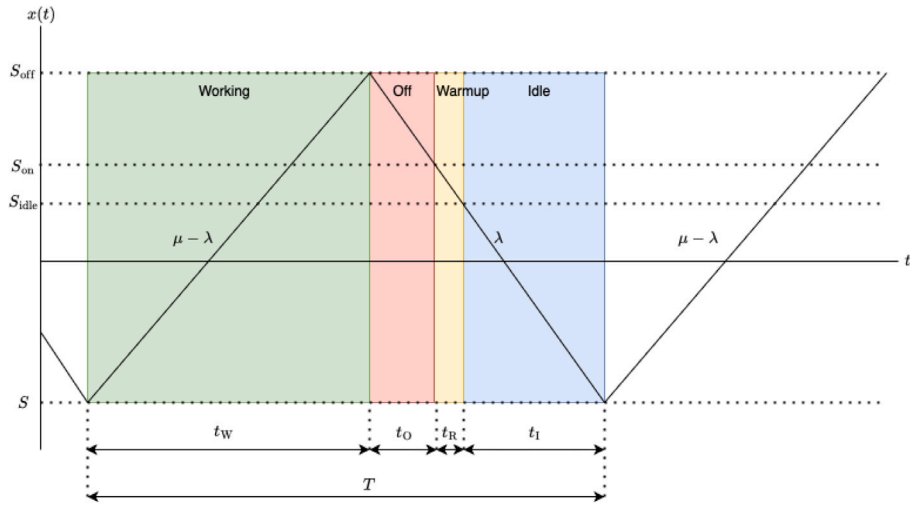

Fig. C.5. Inventory/Backlog dynamics under Working-Off-Idle Policy.

$\frac{S_{\text{off}}-S}{\mu-\lambda}$, $t_{\text{O}} = \frac{S_{\text{off}}-S_{\text{on}}}{\lambda}$, and $t_{\text{I}} = \frac{S_{\text{idle}}-S}{\lambda}$, respectively. Correspondingly, the length of one on/off/idle cycle is:

$$T = t_{\text{W}} + t_{\text{O}} + t_{\text{R}} + t_{\text{I}} = \frac{(S_{\text{off}} - S)}{\lambda(1-u)}. \tag{C.1}$$

The total per-cycle energy, inventory holding, and backlog cost can be expressed in terms of the thresholds and system parameters as follows:

$$TC_{\text{woi}}(S_{\text{off}}, S_{\text{idle}}, S) = EC_{\text{woi}}(S_{\text{off}}, S_{\text{idle}}, S) + c^{+}X^{+} + c^{-}X^{-}, \tag{C.2}$$

where the energy cost per cycle $EC_{\text{woi}}(S_{\text{off}}, S_{\text{idle}}, S)$ is:

$$EC_{\text{woi}}(S_{\text{off}}, S_{\text{idle}}, S) = \begin{cases} c_{\text{W}}u + c_{\text{O}}(1-u) + \frac{\lambda(1-u)}{S_{\text{off}}-S}\left(\frac{c_{\text{R}}-c_{\text{O}}}{\tau} + \frac{(S_{\text{idle}}-S)(c_{\text{I}}-c_{\text{O}})}{\lambda}\right), & S_{\text{off}} - S_{\text{idle}} \geq \frac{\lambda}{\tau}, \\ c_{\text{W}}u + c_{\text{I}}(1-u), & S_{\text{off}} - S_{\text{idle}} < \frac{\lambda}{\tau}. \end{cases} \tag{C.3}$$

The average inventory and backlog levels, X^{+} and X^{-}, are the same as given in Eq. (2), and they do not depend on the selection of S_{idle}. So, selecting S_{idle} affects only the energy cost.

Proof of Theorem 2. The selection of the thresholds must yield non-negative on, off, and idle periods shorter than the cycle length. Writing these constraints yields the following conditions for the thresholds:

$$S_{\text{off}} \geq S_{\text{idle}} \geq S. \tag{C.4}$$

Since $c_{\text{I}} > c_{\text{O}}$, the total cost given in Eq. (C.2) is an increasing function of S_{idle} when $S_{\text{idle}} \leq S_{\text{off}} - \frac{\lambda}{\tau}$ and constant when $S_{\text{idle}} > S_{\text{off}} - \frac{\lambda}{\tau}$. Therefore, for given optimal values of S^{*}_{off} and S^{*}, minimizing the total cost given in Eq. (C.2) by changing S_{idle} subject to the constraint given in Eq. (C.4) gives either $S^{*}_{\text{idle}} = S^{*}$ or $S^{*}_{\text{off}} \geq S^{*}_{\text{idle}} > S^{*}_{\text{off}} - \frac{\lambda}{\tau}$. Since setting $S^{*}_{\text{idle}} = S^{*}$ is equivalent to the Working-Off policy and setting $S^{*}_{\text{off}} \geq S^{*}_{\text{idle}} > S^{*}_{\text{off}} - \frac{\lambda}{\tau}$ is equivalent to the Working-Idle policy, using a Working-Off or Working-Idle policy is always better than a hybrid Working-Off-Idle Policy. □

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