A Capacitated Mobile Facility Location Problem with Mobile Demand: Recurrent Service Provision to En Route Refugees

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ABSTRACT

In this paper, we help humanitarian organizations provide service via mobile facilities (MFs) to migrating refugees, who attempt to cross international borders. Over a planning horizon, we aim to optimize number and routes and relocations of the MFs over a planning horizon. The problem is represented on a network where several refugee groups relocate in their predetermined paths throughout the periods. To incorporate continuity of service, each refugee group should be served at least once every fixed consecutive periods via capacitated MFs. We aim to minimize the total cost, consisting of fixed, service provision, and MF relocation costs, while ensuring the service continuity requirement. We formulate a mixed integer linear programming (MILP) model for this problem. We develop a matheuristic and an accelerated Benders decomposition algorithm as an exact solution method. The proposed model and solution methods are investigated over instances we extracted from the 2020 Honduras migration crisis.

Keywords: humanitarian logistics; capacitated mobile facility location; mobile demand; en route refugees; Benders decomposition; matheuristic

1 INTRODUCTION

Migration, caused by worldwide economic, political, social, and environmental unfavorable conditions, has become a global phenomenon. It is defined as the movement of people from one place to another with a long-run settling purpose [18]. According to the UN Refugee Agency [31], a refugee is someone who has been forced to leave his or her home country because of violence, famine, or natural disasters. The United Nations High Commissioner for Refugees (UNHCR) reported that about 65.6, 79.5, and 82.4 million individuals were forcibly displaced as refugees during 2017, 2019, and 2020, respectively [30].

Refugees are prone to several health risks. They are highly vulnerable to infectious diseases [32] and are often exposed to traumas due to poor living conditions and forced displacements [8]. According to a survey by Morina et al. [21], post-traumatic stress and anxiety are the most prevalent mental disorders among refugees. Besides psychological problems, refugees also suffer from physical traumas. Comellas et al. [8] emphasizes that the somatic distress associated with functional disabilities warrants more attention in both studies and practice. Clearly, providing medical care, nutrition, and shelter alleviates the difficulties of lengthy and long-lasting displacement for refugees.

In recent years, Mobile Facilities (MFs) have been frequently utilized in providing service for an increasing number of transiting refugees. As an example, we can consider the 2015 refugee crisis, where according to Shortall et al. [25], about 850,000 refugees and asylum seekers moved to Greece in 2015. With the purpose of providing health service, the "Doctors of the World" established the refugee ferry project and provided primary health care on-board a commercial ferry. As another example, Médecins Sans Frontieres (MSF; Doctors Without Borders) opened a mobile clinic on the Serbia-Hungary border and treated almost 100 people each day. This was while about 2,000 people crossed the border every day [13]. These examples underline the significance of providing mobile services such as basic health care and relief item delivery.

In this paper, we focus on the provision of various services including food, medicine, or other relief items to transiting refugees by means of MFs. We aim to support decision making while operating the services efficiently, by optimizing the number, locations, and re-locations of the MFs over time. While doing so, we consider the capacities of the facilities and the service needs as well. We represent the problem on a directed network over several time periods such that the refugee groups entering the network in different periods follow distinct paths defined over the nodes and arcs of the network toward their destination nodes. Meanwhile, the MFs relocate on the nodes to provide services to the refugee groups at the same node and time period, depending on their service capacities. The service provision for refugee groups should be recurrent and the goal is to serve each refugee group at least once every fixed number of consecutive time periods, that is determined by the service provider, in order to maintain continuity of service over time.

In section 2, we provide a review of the related literature. We define the MCM-FLP-MD formally in section 3 and provide a mixed integer linear programming (MILP) model. In section 4, we develop two alternative solution methods for the problem: a Network Decomposition Matheuristic (NDM) and an accelerated Benders decomposition (BD) approach as an exact solution method. Section 5 presents computational results. Finally, section 6 concludes with remarks and future work directions.

2 LITERATURE REVIEW

Our problem falls into the broad category of Facility Location Problems (FLPs). Such a problem decides where to locate facilities and how to allocate clients while minimizing the costs of serving them [19]. According to Farahani and Hekmatfar [10], FLPs have essentially four components: 1) customers, 2) facilities, 3) a network where customers and facilities are located, and 4) a metric that represents the travel time or distance among customers and facilities. Our focus is mainly on the Mobile Facility Location Problem (MFLP) and Mobile Facility Routing Problem (MFRP), where MFs can re-position on a network. The MFLP has increasingly gained researchers' attention mainly because of its applicability in social outreach activities. Recent surveys of MFLP in health care systems and humanitarian logistics include Afshari and Peng [1], Ahmadi-Javid et al. [2], Song et al. [26], and Nasrabadi et al. [22]. The MFRP was addressed by [14] with the aim of maximizing the served demand via mobile fleets over a time horizon. The authors proposed approximate solution methods for their proposed mixed integer program and showed that although the problem is NP-Hard, it is polynomially solvable for the single facility case. Later on, Halper et al. [15] introduced an integer programming formulation for MFLP and suggested a decomposition algorithm for the formulation followed by local neighborhood search heuristics. Most studies of MFLP aim to minimize the total distance traveled by the facilities, while all demand is served [11]. On the other hand, Bélanger et al. [4] discussed recent optimization models for location, relocation, and dispatching of medical MFs such as ambulances. The authors noted that the equity and fairness metrics have recently become of highest importance in this context. Our problem inherently captures these metrics since we define an identical service frequency τ to cover the needs of all refugees uniformly.

Facility capacities have been ignored in most MFLP studies. Raghavan et al. [23] studied the capacitated version of the MFLP (CM-FLP) where facilities may move only once, and clients travel to the facilities. They developed a branch-and-price algorithm and two heuristic algorithms for this problem. Our problem extends this study in terms of multi-period planning and the continuity of service frequency that is explicitly accounted for.

In addition to the mobility of facilities, we focus on the mobility of demand as well. We see that demand mobility has been captured in the flow-interception location problem (FILP) in the literature. In fact, the single-period FILP was initially studied by Hodgson [16, 17] and Berman [6]. The objective of FILP is to find locations for facilities to maximize the coverage of demand flow. Applications of the flow-demand coverage problem lie mostly in urban areas, where providing a service only once is sufficient to cover a customer's requirements, as opposed to our case. Berman [7] studied the FILP for customers who travel in a network, not just for the purpose of receiving service. The author divided the customers into stationary and mobile types and considered the existence of both demand types in their formulations. Zeng et al. [33] introduced an "integer-friendly" generalized formulation for the flow-interception model, which can solve several deterministic flow-interception problems. As an extension, the multi-period FILP was introduced by Sterle et al. [27], where some portable facilities intercept the demand flow, and various objectives such as maximizing the intercepted demand or minimizing the relocation costs are inspected.

Most objective functions focus on minimizing fixed or variable costs, or total distance or travel time, or maximizing total benefit under demand coverage. Also, the mobility of both facilities and demand followed by periodic services has been addressed scarcely in the literature. Among solution techniques, heuristics are prevailing solution methods suggested for the FLP and MFLP; however, we provide an exact solution method in addition to a heuristic solution approach. To the best of our knowledge, the MCM-FLP-MD with periodic service provision is introduced for the first time in this study.

3 PROBLEM DEFINITION

In this section, we provide details of the MCM-FLP-MD by first explaining its key elements, then followed by formulating an MILP model for the problem. This problem is set on a connected graph $G = (V, A)$, where node set V and A indicate locations of interest and the roads connecting them, respectively. Refugee groups follow predetermined paths on the network beginning from a source node and ending at a destination node. Each arc on each path is defined in the following way: it takes only one time period to move along the arc according to a refugee group's transportation mode, i.e., by walking or by a vehicle.

We denote the set of paths by P, which is w.l.o.g. finite. For each path $p \in P$, l_p and n_{pk} represents the number of nodes, and k^{th} node on the path, respectively. For instance, $n_{p,1}$ and n_{p,l_p} denote the origin and destination nodes of path p, respectively. Every period, each refugee group moves from a node to the next node along its path. More than one refugee group may follow the same path by entering the path in different periods.

Refugees typically move in groups, and follow a path that is determined at the beginning of their trip. Refugees who enter the network in the same period and follow the same path are assumed to form one refugee group $r \in R$ in our study. We assume the humanitarian organizations that provide services can predict these paths for planning purposes [12, 28]. We denote the path traversed by some refugee group $r \in R$ by $p_r \in P$, and the time period when they enter the path by $e_r \in T$, where T is the set of periods on the planning horizon and each time period can stand for a single day. The length of the planning horizon is determined such that all refugee groups will leave the network by period $|T|+1$. Considering the path lengths and time periods when refugee groups enter the network, we note that it is important to determine $|T|$ precisely to avoid unnecessary variables. Accordingly, we use Equation (1) to calculate $|T|$.

$$T = \{1, ..., |T|\}, \quad |T| = \max_{r \in R}\{l_{p_r} + e_r - 1\} \tag{1}$$

Let d_r represent the demand level of refugee group $r \in R$, which shows their population size. To guarantee demand satisfaction under limited capacities of MFs, a refugee group can be served simultaneously by multiple MFs at a node. For continuity of service, the whole population of each refugee group must be served at least once and could partially be served several times every consecutive τ periods.

M represents the set of available MFs. Each MF $m \in M$ has a capacity Δ_m, indicating maximum number of refugees that facility m can serve in a single period. Not all MFs need to be used in a solution. Thus, $|M|$ is an upper bound on the number of required MFs and the model decides the number of MFs to be utilized based on service requirements. Recruited MFs can enter the network at any time. The entrance point of an MF is the first node where it should provide service at.

State of being at node $i \in V$ in period $t \in T$ is represented by "(i,t) node-time pair" for both refugees and MFs. The process of providing service via an MF $m \in M$ at node-time pair (i,t) is referred to as a 'service act'. A solution to the problem consists of a list of service acts for each recruited MF. In each time period, Every MF can just show up at one node-time pair and if it provides service at that node, it incurs a service act cost. Indeed, each service act of an MF occurs by the presence of that MF and at least one refugee group at a specific node-time pair. Finally, the objective function of this problem comprises three terms: 1) the total fixed cost of utilizing the MFs, 2) the total operating costs associated with the service acts, and 3) the total transportation cost associated with the relocation of MFs on the network.

3.1 Mathematical Model

We propose the following MILP to formulate the MCM-FLP-MD.

Sets:

V	Set of nodes
P	Set of paths
R	Set of refugee groups entering the network
M	Set of potential mobile facilities
T	Set of time periods

Parameters:

d_r	Population of refugee group $r \in R$
p_r	Path traversed by refugee group $r \in R$ ($p_r \in P$)
e_r	Time period in which refugee group $r \in R$ enters path p_r ($e_r \in T$)
l_p	Number of nodes on path $p \in P$
n_{pk}	k^{th} node on path $p \in P$ where $k = 1, ..., l_p$
Δ_m	Service capacity for mobile facility $m \in M$
f_m	Fixed cost of using mobile facility $m \in M$
o_m	Service act operating cost for mobile facility $m \in M$
c_{ij}	Traveling cost from node $i \in V$ to node $j \in V$
τ	Service frequency level in terms of number of time periods

Decision Variables:

A_{irt}	Fraction of population of refugee group $r \in R$ who are planned to receive service at node $i \in V$ in period $t \in T$
S_{imt}	1, if mobile facility $m \in M$ provides service at node $i \in V$ in period $t \in T$; 0, otherwise
X_{ijmt}	1, if mobile facility $m \in M$ is located at node $i \in V$ in period $t \in T$ and at node $j \in V$ in period $t+1$; 0, otherwise (Since there is no refugee group in the network in $t = |T|+1$, MFs remain at their nodes in between t and $t+1$ where $t = |T|$)
Y_m	1, if mobile facility $m \in M$ is used; 0, otherwise

MILP Mathematical Model:

$$\min \sum_{m \in M}(f_m Y_m + \sum_{i \in V}\sum_{t \in T} o_m S_{imt} + \sum_{i \in V}\sum_{j \in V}\sum_{t \in T} c_{ij} X_{ijmt}) \tag{2}$$

subject to

$$\sum_{t'=0}^{\tau-1} A_{(n_{p_r,q+t'+1}),r,(e_r+q+t')} \geq 1 \qquad \forall r \in R, 0 \leq q \leq l_{p_r} - \tau \tag{3}$$

$$\sum_{m \in M} \Delta_m S_{imt} \geq \sum_{r \in R} d_r A_{irt} \qquad \forall i \in V, t \in T \tag{4}$$

$$S_{imt} \leq \sum_{j \in V} X_{ijmt} \qquad \forall i \in V, m \in M, t \in T \tag{5}$$

$$\sum_{j \in V} X_{jimt} \leq \sum_{j \in V} X_{ijm(t+1)} \qquad \forall i \in V, m \in M, t = 1, ..., |T|-1 \tag{6}$$

$$\sum_{i \in V}\sum_{j \in V} X_{ijmt} \leq Y_m \qquad \forall m \in M, t \in T \tag{7}$$

$$A_{irt} \geq 0 \qquad \forall i \in V, r \in R, t \in T \tag{8}$$

$$S_{imt} \in \{0,1\} \qquad \forall i \in V, m \in M, t \in T \tag{9}$$

$$X_{ijmt} \in \{0,1\} \qquad \forall i, j \in V, m \in M, t \in T \tag{10}$$

$$Y_m \in \{0,1\} \qquad \forall m \in M \tag{11}$$

The objective function (2) minimizes the total costs consisting of the MF establishment costs, service act costs and relocation costs. Constraints (3) guarantee that each refugee group is served at least once, and could partially be served multiple times during consecutive τ periods. Constraints (4) ensure that the total capacity provided by MFs at each node-time pair satisfies the fractional demand of refugee groups who are planned to receive service. Constraints (5) indicate that an MF can provide service at a node-time pair only if it is located there. Constraints (6) define flow conservation of MFs among the nodes of the network from one period to the next. Constraints (7) determine whether an MF is used. Finally, constraints (8)-(11) define the domains of the decision variables.

4 SOLUTION METHODS

In this section, we introduce two solution methods to solve the MCM-FLP-MD. We first develop a Network Decomposition Matheuristic (NDM), which requires relatively short run time compared to run times of our MILP formulation. Second, we develop an accelerated Benders decomposition (BD) algorithm as an exact solution method. We describe the NDM and the BD in detail in sections 4.1 and 4.2, respectively.

4.1 Network Decomposition Matheuristic

The Network Decomposition Matheuristic (NDM) algorithm aims to manage the complexity associated with variables having multiple indices by decomposing the problem based on paths. Each path $p \in P$ traversed by some refugee groups, corresponds to a smaller network and forms a subproblem for MCM-FLP-MD, referred to as (SP^p). Instead of solving the problem over the entire network, NDM solves $n \leq |P|$ subproblems independently, each having considerably tighter solution space. Results of each subproblem creates a partial solution to the original problem, independent of other subproblems. Aggregation of all solution pieces forms the solution associated with the entire network.

For each subproblem (SP^p), R^p denotes the set of refugee groups that migrate along path p. Also, the set of MFs required for serving R^p on path $p \in P$ is referred to as M^p and the minimum number of these MFs is calculated by Equation (12).

$$|M^p| = \left\lceil \frac{\sum_{r \in R^p} d_r}{\Delta\tau} \right\rceil \tag{12}$$

While solving the SP^p, insufficient $|M^p|$ may lead to infeasible solutions. In such cases, we can simply increment $|M^p|$ by one unit and solve the (SP^p) again until we reach a feasible solution. The planning horizon for each subproblem is kept the same as that of the original problem to facilitate the consolidation of the subproblem solutions. The mathematical model corresponding to subproblem SP^p given below, is a simplified version of the MILP for the original problem where the Y_m variables are excluded. By defining the sets and parameters independently for each path, the problem size is reduced significantly.

New Sets:

V^p	Set of nodes of the network that lie on path $p \in P$
R^p	Set of refugee groups entering path $p \in P$
M^p	Set of recruited mobile facilities to provide service along path $p \in P$

Mathematical Model:

$$\min \sum_{i \in V^p} \sum_{m \in M^p} \sum_{t \in T} o_m S_{imt} + \sum_{i \in V^p} \sum_{j \in V^p} \sum_{m \in M^p} \sum_{t \in T} c_{ij} X_{ijmt} + \sum_{m \in M^p} f_m \tag{13}$$

subject to

$$\sum_{t'=0}^{\tau-1} A_{(n_{p,q+t'+1}),r,(e_r+q+t')} \geq 1 \qquad \forall r \in R^p, 0 \leq q \leq l_p - \tau \tag{14}$$

$$\sum_{m \in M^p} \Delta_m S_{imt} \geq \sum_{r \in R^p} d_r A_{irt} \qquad \forall i \in V^p, t \in T \tag{15}$$

$$S_{imt} \leq \sum_{j \in V^p} X_{ijmt} \qquad \forall i \in V^p, m \in M^p, t \in T \tag{16}$$

$$\sum_{j \in V^p} X_{jimt} \leq \sum_{j \in V^p} X_{ijm(t+1)} \qquad \forall i \in V^p, m \in M^p, t = 1, ..., |T| - 1 \tag{17}$$

$$A_{irt} \geq 0 \qquad \forall i \in V^p, r \in R^p, t \in T \tag{18}$$

$$S_{imt} \in \{0, 1\} \qquad \forall i \in V^p, m \in M^p, t \in T \tag{19}$$

$$X_{ijmt} \in \{0, 1\} \qquad \forall i, j \in V^p, m \in M^p, t \in T \tag{20}$$

The NDM procedure is described in detail in Algorithm 1. In this algorithm, we initially set both the number of MFs and the overall objective value to 0. Then, for each path $p \in P$, we determine the refugee groups traversing path p and calculate the required MFs to serve on path p based on Equation (12). After solving the subproblem SP^p, we update the list of MFs and the objective value of the problem solved up to that point.

Algorithm 1 NDM Framework

N_{MF}^{NDM},$Z^{NDM} \leftarrow 0$. // Total number of MFs to be used in the final solution and final objective function value.
for $p \in P$ **do**:
  if $p \in \{p_r;\ \forall r \in R\}$ **then**: // Checking if the path p is traversed by refugee groups. If true, we solve SP.
    $R^p \leftarrow \{r \in R : p_r = p\}$ // Defining R^p.
    Calculate $|M^p|$ using Equation (12). // Calculating number of MFs to be used in path p.
    while True **do**: // Checking feasibility of the solution of SP^p considering $|M^p|$.
      $M^p \leftarrow \{N_{MF}^{NDM} + 1, ..., N_{MF}^{NDM} + |M^p|\}$. // Updating the elements of set M^p to differentiate the MFs of subproblems.
      Solve MILP for the subproblem SP^p and obtain $Z(SP^p)$ if SP^p is feasible.
      if SP^p is infeasible **then**:
        $|M^p| \leftarrow |M^p| + 1$.
        continue
      end if
      break
    end while
    $N_{MF}^{NDM} \leftarrow N_{MF}^{NDM} + |M^p|$ // Updating the number of MFs used up to this iteration.
    $Z^{NDM} \leftarrow Z^{NDM} + Z(SP^p)$ // Updating the costs of the network up to this iteration.
  end if
end for
Output: N_{MF}^{NDM}, Z^{NDM} // Return the number of MFs and the objective value of the solution of NDM.

4.2 Benders Decomposition Algorithm

The Benders Decomposition (BD) method was introduced in the early 1960s as a partition-based solution strategy for large-scale MILPs [5]. BD is successfully applied in diverse fields [24]. In the field of transportation, Costa [9] presented a review of BD applications on network design problems, where integer and continuous variables are mainly associated with arc selection and commodity flow amounts, respectively. The author indicated that BD outperforms some traditional techniques such as Branch-and-Bound or Lagrangian Relaxation for network design problems because of their special structure.

In BD, the problem is divided into a restricted master problem (RMP) and a linear subproblem (LSP). The RMP consists of constraints that contain pure integer variables. The LSP, is obtained via fixing the values of integer variables based on the solution of the RMP. Iteratively, the solution of the RMP is used in the dual of the LSP, referred to as dual subproblem (DSP) and the solution of the DSP generates Benders feasibility or optimality cuts for the RMP. This procedure is continued until a stopping criterion is met. The LSP, DSP, and RMP models associated with our mathematical model are introduced next.

LSP: (Contains the continuous decision variables A_{irt})

$$\min \quad 0 \tag{21}$$

subject to

$$\sum_{t'=0}^{\tau-1} A_{(n_{p_r,q+t'+1}),r,(e_r+q+t')} \geq 1 \qquad \forall r \in R, 0 \leq q \leq l_{p_r} - \tau \tag{22}$$

$$\sum_{m\in M} \Delta_m \overline{S}^{\theta}_{imt} \geq \sum_{r\in R} d_r A_{irt} \qquad \forall i \in V, t \in T \tag{23}$$

$$A_{irt} \geq 0 \qquad \forall i \in V, r \in R, t \in T \tag{24}$$

In the LSP, $\overline{S}^{\theta}_{imt}$ are defined to be equal to the values of S_{imt} variables of the RMP at iteration θ. Since the A_{irt} do not contribute to the objective function of the problem, the objective function of the LSP is set to 0.

Dual Decision Variables: (Corresponding to const. (22) and (23))

$$u_{rq} \qquad \forall r \in R, q = 0, ..., l_{p_r} - \tau$$
$$v_{it} \qquad \forall i \in V, t \in T$$

DSP: (Dual of the LSP model)

$$\max \sum_{r\in R} \sum_{q=0,...,l_{p_r}-\tau} u_{rq} - \sum_{i\in V} \sum_{m\in M} \sum_{t\in T} \Delta_m \overline{S}^{\theta}_{imt} v_{it} \tag{25}$$

subject to

$$\sum_{max\{0,k-\tau\}\leq q\leq min\{k-1,l_{p_r}-\tau\}} u_{rq} - d_r v_{n_{p_r k},e_r+k-1} \leq 0 \qquad \forall r \in R,\ k = 1, ..., l_{p_r} \tag{26}$$

$$u_{rq} \geq 0 \qquad \forall r \in R, q = 0, ..., l_{p_r} - \tau \tag{27}$$

$$v_{it} \geq 0 \qquad \forall i \in V, t \in T \tag{28}$$

We refer to the objective function of the DSP at iteration θ as W^{θ}_{DSP}.

RMP: (Consists of pure binary variables and $\eta \geq 0$)

$$\min \sum_{m\in M} f_m Y_m + \sum_{i\in V} \sum_{m\in M} \sum_{t\in T} o_m S_{imt} + \sum_{i\in V} \sum_{j\in V} \sum_{m\in M} \sum_{t\in T} c_{ij} X_{ijmt} + \eta \tag{29}$$

subject to

$$S_{imt} \leq \sum_{j\in V} X_{ijmt} \qquad \forall i \in V, m \in M, t \in T \tag{30}$$

$$\sum_{j\in V} X_{jimt} \leq \sum_{j\in V} X_{ijm(t+1)} \qquad \forall i \in V, m \in M, t = 1, ..., |T| - 1 \tag{31}$$

$$\sum_{i\in V} \sum_{j\in V} X_{ijmt} \leq Y_m \qquad \forall m \in M, t \in T \tag{32}$$

$$S_{imt} \in \{0, 1\} \qquad \forall i \in V, m \in M, t \in T \tag{33}$$

$$X_{ijmt} \in \{0, 1\} \qquad \forall i, j \in V, m \in M, t \in T \tag{34}$$

$$Y_m \in \{0, 1\} \qquad \forall m \in M \tag{35}$$

$$\eta \geq 0 \tag{36}$$

The feasibility and optimality cuts are incorporated into the RMP throughout the iterations according to Equations (37) and (38).

$$0 \geq \sum_{r\in R} \sum_{q=0,...,l_{p_r}-\tau} \overline{u}^{\theta}_{rq} - \sum_{i\in V} \sum_{m\in M} \sum_{t\in T} \Delta_m S_{imt} \overline{v}^{\theta}_{it} \qquad \text{"feasibility cut"} \tag{37}$$

$$\eta \geq \sum_{r\in R} \sum_{q=0,...,l_{p_r}-\tau} \overline{u}^{\theta}_{rq} - \sum_{i\in V} \sum_{m\in M} \sum_{t\in T} \Delta_m S_{imt} \overline{v}^{\theta}_{it} \qquad \text{"optimality cut"} \tag{38}$$

Proposition 4.1. *The optimal value for η is 0.*

Proof. Since the optimal objective value of the LSP is 0, according to the duality theory, optimal objective value of DSP must be 0. This implies that we can directly 0 to the variable η. □

Proposition 4.2. *Starting from any feasible solution, if $W^{\theta}_{DSP} = 0$ at an iteration θ, then the stopping criterion for our BD algorithm is met and the solution of RMP in the corresponding iteration is optimal solution of the original problem.*

Proof. Z^{RMP} provides a lower bound (LB) for the optimal objective value. The term $c^T y + W^{\theta}_{DSP}$ (where y and c^T represent the binary variables (S, X, Y) and their coefficients in the objective function of the original problem, respectively), provides an upper bound (UB). The algorithm stops when $LB = UB$. We showed in Proposition 4.1 that the optimal value for variable η is 0. Letting $\eta = 0$ implies $Z^{RMP} = c^T y$. Therefore, we conclude that $LB = UB$ and the algorithm stops, if $W^{\theta}_{DSP} = 0$ is obtained at any iteration θ. □

4.3 Improvements for the proposed BD algorithm

Although BD benefits a powerful theory, the straightforward application of classical BD is usually slow in convergence. In this section, we applied two refinements on the proposed BD.

(1) **Multi-cut implementation:** At each iteration of the classic BD, a single Benders cut is inserted to the RMP. However, the multi-cut reformulation outperforms the single-cut approach as it strengthens the RMP more quickly [24]. So, we generate multiple Benders cuts by solving DSP several times at a single iteration and insert those cuts simultaneously into the RMP.

(2) **Linear RMP Relaxation:** Solving the RMP is usually time consuming and solving it to optimality in the initial iterations is not necessary. McDaniel and Devine [20] showed that valid Benders cuts can be generated by the solutions to the LP relaxation of RMP. We solve the linear relaxation of the RMP (LR-RMP) at early iterations of the algorithm. By applying this approach, the RMP is enriched with high-quality Benders cuts.

5 COMPUTATIONAL ANALYSIS

We implemented the MILP formulation and solution methods on the real-life migration crisis, took place in Honduras in late 2020, when groups of refugees began migrating from Central America, with the hope of reaching Mexico and the USA, often on foot and in groups known as “caravans". Guatemalan migration officials estimated that about 6,000 migrants, most of them Honduran, were corralled between Chiquimula and the border with Honduras. Another caravan of about 4,000 refugees, mostly Honduran migrants, had camped out near the village of Vado Hondo in Guatemala. The migrants were traveling by a combination of walking, hitchhiking, and bus [3, 29].

In section 4, we proposed two solution methods for the MCM-FLP-MD. In addition to these solution methods, we directly solved the instances via the proposed MILP formulation. We implemented the MILP formulation and the two proposed solution methods in the Python programming environment, Spyder Anaconda IDE 4.1.5 platform and solved them using the Pyomo optimization package and the solver CPLEX 12.10.0.. All experiments are conducted on a workstation with 64-bit operating system, Xenon(R) 2.60 GHz

CPU and 128 GB of RAM (18 cores and 36 processors). Considering instance complexities and solution specifications, 0.01% and 0.02% optimality gaps are allowed for the medium- and large-sized instances. Moreover, all instances are solved with a 6-hour (21600s) time limit. Due to relatively short run times associated with small-sized instances, we applied the accelerated BD procedure only to medium- and large-sized instances.

According to Table 1, the MILP formulation is preferred for small-sized networks with approximately 20 nodes and a time horizon of about two weeks as it achieves optimal solutions quickly. Also, the accelerated BD is preferred for medium-sized networks comprised of approximately 30 nodes and with a time horizon of about 3 weeks. Finally, the NDM is preferred when the instance sizes are large, consisting of about 50 nodes or more and lasting for more than a month. Finally, we observed a 2.6% objective value gap between the NDM and MILP results among all 60 instances.

Instance set	**Average run-time**			**Average Obj. value ratio**			**Solver mip Gap %**	
	NDM	MILP	BD	NDM/MILP	NDM/BD	MILP/BD	MILP	BD
Small	24.8	118.2	-	1.017	-	-	-	-
Medium	152.5	996.4	624.1	1.031	1.031	1	-	-
Large	856.2	19095.2	12919.2	1.027	1.036	1.009	1.55	0.71

Table 1: Overall results corresponding to instance sets

6 CONCLUSION

In this paper, we studied a multi-period capacitated mobile facility location problem with mobile demands (MCM-FLP-MD). This problem aims to provide recurrent humanitarian aid to en route refugee groups during their migration in an effective manner using capacitated Mobile Facilities (MFs). We proposed an MILP formulation for the problem followed by two solution methods: a Network Decomposition Matheuristic (NDM) and an accelerated Benders decomposition (BD) approach as an exact solution method. Our observations indicated that regarding tun times, the MILP formulation, accelerated BD and NDM algorithm are most suitable for solving small, medium, and large-sized instances, respectively.

A future research direction for this problem is to further improve both the MILP model by incorporating suitable valid inequalities, and the NDM algorithm in order to better utilize capacities of MFs in the network. Also, incorporating uncertainties in the network, especially regarding the predetermined paths assigned to the refugees and their displacement patterns is another research direction for which stochastic dynamic programming can be investigated. Constructive heuristics and metaheuristic solution methods may also be applicable for extensions of this problem.

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