Available online at www.sciencedirect.com
ScienceDirect
IFAC PapersOnLine 52-13 (2019) 1931–1936

A Hierarchical Approach for Solving Simultaneous Lot Sizing and Scheduling Problem with Secondary Resources

Cevdet Utku Şafak*, Görkem Yılmaz, Erinç Albey*****

** Department of R&D, Vestel Electronics 45030, Manisa, Turkey (e-mail: cevdet.safak@vestel.com.tr)*
*** Department of Industrial Engineering, Ozyegin University, 34794, Istanbul, Turkey (e-mail: gorkem.yilmaz@ozyegin.edu.tr)*
**** Department of Industrial Engineering, Ozyegin University, 34794, Istanbul, Turkey (e-mail: erinc.albey@ozyegin.edu.tr)*

Abstract: This study represents a decomposition heuristic approach for simultaneous lot sizing and scheduling problem for multiple product, multiple parallel machines with secondary resources. The motivation of the study comes from the real-world instance of a plastic injection plant at Vestel Electronics. The plastic injection plant requires plastic injection molds at the planner's disposal, in order to produce variations of products, by the compatible plastic injection machines. The variations on the molds and the mold changes on the machines bring out sequence dependent major and minor setups. Since each machine requires an operator, we have extended the formulation with workforce and shift planning. Results show that proposed heuristic yields comparable solutions to that of exact model for small and medium size instances; and provides schedules for the large size instances, for which exact model cannot find a feasible solution in the allotted time.

© 2019, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Simultaneous lot sizing and scheduling, decomposition heuristics, sequence dependent setups, secondary resources, workforce planning, shift planning.

1. INTRODUCTION

Integrating operational problems and developing solution strategies that tackle these problems simultaneously have recently gained the attention of both researchers and practitioners, Copil et al. (2016). The lot sizing decisions are made in order to economically balance the variations of the demand by balancing inventory and setup costs and, scheduling decisions define the sequence of production orders of different products in the planning horizon. For the problems, where sequence dependent setups matter, it is required to make lot sizing and scheduling decisions simultaneously. The simultaneous lot sizing and scheduling models are classified as large bucket and small bucket models. The large bucket models use macro-periods and small bucket models consider micro-periods to sequence the products within the planning periods under resource capacity constraints. We provide a reformulation of the capacitated lot sizing problem with sequence-dependent setups (CLSD), which is presented in Haase (1996). This approach decomposes the problem into macro-periods and the sequence of the products are formulated as it is in a travelling salesman problem (TSP).

Kwak and Jeong (2011) proposes a two stage hierarchical approach to single machine multi product CLSD. In the first stage, capacitated lot sizing problem (CLP) is solved. In the second stage, lot sizes found in the CLP problem are fixed and the lower level scheduling problem is solved by minimizing the make span of the production. Meyr and Mann (2013) propose a hierarchical solution approach for large scale parallel non-identical machine general lot sizing and scheduling problem (GLSP) with sequence dependent setups by decomposing the problem into non-identical sets of single machine problems. The decomposition assigns demand and initial inventories to the single machines and then the parallel problems are solved iteratively. The presented heuristics aims to find the best decomposition, which gives feasible, and potentially, the optimal solution. Tavaghof-Gigloo et al. (2016) study workforce planning and the effect of flexible shift and overtime deviations on the overall performance of the production plans. Their study considers lot sizing but does not include the scheduling decisions for the production plans. Hemig et al. (2014) studies integrated production and staff planning problem in automotive industry, which does not consider the setup times and costs.

In this work, we consider the scheduling problem of Vestel Electronics' plastic injection plant. Vestel Electronics is one of the largest TV manufacturers in the World and largest in Europe producing over 10 million TV sets annually. A typical TV consist of electronics, metal and plastic parts. This paper concentrates on the plastic parts production plant, which operates on tool machine pairs, require setup times in order to change the tools on the machines. In the plastic injection plant we consider, the plastic injection molds are mounted on the machines and production is made by injecting the plastic raw material into the mold. A typical plastic injection mold has version inserts which are interchangeable and via these interchanges, it is possible to produce products with different

2405-8963 © 2019, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2019.11.485

attributes. The interchange of the molds and the versions introduce major and minor sequence dependent setups to the problem.

As a matter of course, the shift plan may change according to the seasonal demand changes and/or sudden increases in the demand of specific products. These changes affect the capacity requirements, which in turn affect the workforce and shift planning decisions. In our work we extend the CLSD formulation to include workforce and shift planning decisions; as well as to cover secondary resources like molds. Extended formulation is good at solving small instances, however to solve larger instances, a hierarchical decomposition heuristic is proposed. The proposed method shows promising results compared to the exact method especially for larger size instances.

The rest of the paper is organized as follows: Section 2 presents the notation and exact formulation used in the study. Section 3 describes the details of the proposed decomposition heuristic. Numerical results are discussed in Section 4 and concluding remarks are presented in Section 5.

2. EXACT FORMULATION AND EXTENSIONS WITH SECONDARY RESOURCES

1.1 Parameters and Decision Variables of the Problem

The notation for parameters used in the exact CLSD formulation is given below:

S^T	Set of periods
S^L	Set of machines
S^M	Set of molds
S^I	Set of products
M	A big number set to the maximum number of products that can be produced in one period
B_i^0	Initial backlog for product *i* ($\forall i \in S^I$)
B_m^0	Total initial backlog of the set of products mold *m* is capable to produce ($\forall m \in S^M$)
S_i^0	Initial inventory for product *i* ($\forall i \in S^I$)
S_m^0	Total initial inventory of the set of products mold *m* is capable to produce ($\forall m \in S^M$)
C_l^P	Unit production cost of machine *l* ($\forall l \in S^L$)
C_i^H	Unit holding cost of product *i* ($\forall i \in S^I$)
C_m^{HM}	Unit holding cost of the set of products mold m is capable to produce ($\forall m \in S^M$)
C_i^B	Unit backlogging cost of product *i* ($\forall i \in S^I$)
C_m^{BM}	Unit backlogging cost of the set of products mold *m* is capable to produce ($\forall m \in S^M$)
C^{LAB}	Cost of labor per worker
C^{OT}	Cost of overtime per worker
C^{ST}	Setup cost per second
D_{it}	Demand for product *i* at period *t* ($\forall i \in S^I$; $\forall t \in S^T$)
D_{mt}	Total demand for the set of products mold *m* is capable to produce ($\forall m \in S^M$)
D_{it}^W	1 *iff* a demand is present to mold *m* at period *t* ($\forall m \in S^M$; $\forall t \in S^T$)
T_{ij}^{ST}	Set up time from product *i* to product *j* ($\forall i \in S^I$; $\forall j \in S^I$)
T_m^{STM}	Set up time for mold *m* ($\forall m \in S^M$)
T_i^{CYC}	Cycle time for product *i* ($\forall i \in S^I$)
T_m^{CYCM}	Cycle time for product i ($\forall m \in S^M$)
T^C	Total available production time in one shift
Q^W	Number of available operators for the shifts per day
Q^{OT}	Number of available workers for overtime
Q_m^M	Mold quantity for mold m ($\forall m \in S^M$)
Q_m^P	Number of products mold m is capable to produce ($\forall m \in S^M$)
Ω_{mi}	1 *iff* mold m is capable to produce product *i*, 0 otherwise ($\forall m \in S^M$;$\forall i \in S^I$)
Γ^T	Percentage of increase in a shift duration if overtime is made

The notation for the decision variables is presented below:

z_{ijlt}	1 *iff* machine *l* produces product *i* and *j* consecutively at period *t* ($\forall i \in S^I \cup \{0\}$; ($\forall j \in S^I \cup \{0\}$; $\forall l \in S^L$; $\forall t \in S^T$)
w_{ilt}^I	1 iff machine l produces product *i* at period *t* ($\forall i \in S^I \cup \{0\}$;$\forall l \in S^L$; $\forall t \in S^T$)
w_{mlt}^M	1 *iff* mold m is setup at machine *l* at period *t* ($\forall m \in S^M$;$\forall l \in S^L$;$\forall t \in S^T$)
q_{ilt}^I	Production quantity for product *i* produced at machine *l* at period *t* ($\forall i \in S^I$;$\forall l \in S^L$;$\forall t \in S^T$)
q_{mlt}^M	Total production quantity of products, mold *m* produced, at machine *l* at period *t* ($\forall m \in S^M$;$\forall l \in S^L$;$\forall t \in S^T$)
s_{it}^I	Inventory for product *i* produced at machine *l* at period *t* ($\forall i \in S^I$;$\forall t \in S^T \cup \{0\}$)
s_{mt}^M	Total inventory of the products that mold m is capable to produce at machine *l* at period *t* ($\forall m \in S^M$;$\forall t \in S^T \cup \{0\}$)
b_{it}^I	Backlog for product *i* produced at machine *l* at period *t* ($\forall i \in S^I$;$\forall t \in S^T \cup \{0\}$)
b_{mt}^M	Total backlog of the products that mold m is capable to produce at machine *l* at period *t* ($\forall m \in S^M$;$\forall t \in S^T \cup \{0\}$)
u_{ilt}	Additional variable for sub tour eliminations ($\forall i \in S^I$;$\forall l \in S^L$; $\forall t \in S^T$)
μ_{lt}^I	1 *iff* first shift is made on machine *l* at period *t* ($\forall l \in S^L$; $\forall t \in S^T$)
μ_{lt}^{II}	1 *iff* second shift is made on machine *l* at period *t* ($\forall l \in S^L$; $\forall t \in S^T$)
μ_{lt}^{III}	1 *iff* third shift is made on machine *l* at period *t* ($\forall l \in S^L$; $\forall t \in S^T$)
λ^I	Number of workers on first shift
λ^{II}	Number of workers on second shift
λ^{III}	Number of workers on third shift
β_{lt}	Overtime decision on machine *l* at period *t*

1.2 Exact Mathematical Formulation

The mathematical model formulation, objective function and constraints; along with their detailed explanations are presented below:

$$[MIN]\ Z = \sum_{i\epsilon S^i}\sum_{t\epsilon S^t} C_i^H s_{it}^I + \sum_{i\epsilon S^i}\sum_{t\epsilon S^t} C_i^B b_{it}^I + \sum_{l\epsilon S^l}\sum_{i\epsilon S^i}\sum_{t\epsilon S^t} C_i^P s_{it}^I + \sum_{i\epsilon S^i}\sum_{j\epsilon S^i}\sum_{l\epsilon S^l}\sum_{t\epsilon S^t} C^{ST} T_{ij}^{ST} z_{ijlt} + C^{LAB}(\lambda^I + \lambda^{II} + \lambda^{III}) + \sum_{l\epsilon S^l}\sum_{t\epsilon S^t} C^{OT}\beta_{lt} \tag{1}$$

$$D_{it} + s_{it}^I + b_{it-1}^I = s_{it-1}^I + b_{it}^I + \sum_{l\epsilon S^L} q_{ilt}^I \quad (\forall i \in S^I; \forall t \in S^T) \tag{2}$$

$$M w_{ilt}^I \geq q_{ilt} \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{3}$$

$$\sum_{i\epsilon S^I} T_i^{CYC} q_{ilt} + \sum_{i\epsilon S^I}\sum_{j\epsilon S^I} T_{ij}^{ST} z_{ijlt} \leq T^C(\mu_{lt}^I + \mu_{lt}^{II} + \mu_{lt}^{III} + \Gamma^T \beta_{lt}) \quad (\forall l \in S^L; \forall t \in S^T) \tag{4}$$

$$\sum_{x\epsilon S^I \cup \{0\}} z_{ixlt} = \sum_{y\epsilon S^I \cup \{0\}} z_{yilt} \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{5}$$

$$\sum_{j\epsilon S^I \cup \{0\}} z_{ijlt} \leq w_{ilt}^I \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{6}$$

$$\sum_{i\epsilon S^I \cup \{0\}} z_{i0lt} \leq \mu_{lt} \quad (\forall l \in S^L; \forall t \in S^T) \tag{7}$$

$$\sum_{j\epsilon S^I \cup \{0\}} z_{0jlt} \leq \mu_{lt} \quad (\forall l \in S^L; \forall t \in S^T) \tag{8}$$

$$u_{ilt} \leq |S^I| \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{9}$$

$$u_{ilt} - u_{jlt} + |S^I| z_{ijlt} \leq |S^I| - 1 \quad (\forall i \in S^I; \forall j \in S^I | i \neq j; \forall l \in S^L; \forall t \in S^T) \tag{10}$$

$$b_{it}^I = B_i^0 \ (\forall i \in S^I; t = 0) \tag{11}$$

$$s_{it}^I = S_i^0 \ (\forall i \in S^I; t = 0) \tag{12}$$

$$w_{mlt}^M \leq \mu_{lt}^I \ (\forall m \in S^M; \forall l \in S^L; \forall t \in S^T) \tag{13}$$

$$\mu_{lt}^I \leq \mu_{kt}^I \ (\forall l \in S^L; \forall k \in S^L | k > l; \forall t \in S^T) \tag{14}$$

$$\mu_{lt}^I \geq \mu_{lt}^{II} \quad (\forall l \in S^L; \forall t \in S^T) \tag{15}$$

$$\mu_{lt}^{II} \geq \mu_{lt}^{III} \quad (\forall l \in S^L; \forall t \in S^T) \tag{16}$$

$$\mu_{lt}^I \geq \beta_{lt} \quad (\forall l \in S^L; \forall t \in S^T) \tag{17}$$

$$\mu_{lt}^I + \mu_{lt}^{II} + \mu_{lt}^{III} + \beta_{lt} \leq 3 \ (\forall l \in S^L; \forall t \in S^T) \tag{18}$$

$$\lambda^I + \lambda^{II} + \lambda^{III} \leq Q^W \tag{19}$$

$$\sum_{l\epsilon S^L} \mu_{lt}^I \leq \lambda^I \quad (\forall t \in S^T) \tag{20}$$

$$\sum_{l\epsilon S^L} \mu_{lt}^{II} \leq \lambda^{II} \quad (\forall t \in S^T) \tag{21}$$

$$\sum_{l\epsilon S^L} \mu_{lt}^{III} \leq \lambda^{III} \quad (\forall t \in S^T) \tag{22}$$

$$\sum_{l\epsilon S^L}\sum_{t\epsilon S^T} \beta_{lt} \leq Q^{OT} \tag{23}$$

$$\sum_{l\epsilon S^L} w_{mlt}^M \leq Q_m^M \quad (\forall m \in S^M; \forall t \in S^T) \tag{24}$$

$$\sum_{m\epsilon S^M} \Omega_{mi}\, w_{mlt}^M \geq w_{ilt}^I \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{25}$$

$$z_{ijlt}, w_{ilt}^I, w_{mlt}^M, \mu_{lt}^I, \mu_{lt}^{II}, \mu_{lt}^{III}, \beta_{lt} \in \{0,1\}$$

$$q_{ilt}^I, s_{it}^I, b_{it}^I \in \mathbb{R}^+$$

$$u_{ilt}, \lambda^I, \lambda^{II}, \lambda^{III} \in \mathbb{Z}^+$$

Equation (1) is the objective function including the production, backlogging, inventory holding, setup, labour and overtime costs. Equation (2) is the inventory balance equation. Equation (3) prevents the production of product i if setup is not executed. M is set as the maximum number of products that can be produced in a period. Equation (4) is the capacity constraint for the machines. Equation (5) and (6) guarantee there is only one setup arc coming form (or to) a product if that product is selected to be produced and system is setup accordingly. Equation (7) and (8) assure there is no setup arc coming from (or to) the dummy product 0 when there is no shift on the machine. Equation (9) and (10) are the sub tour elimination constraints shown in Miller et al. (1960). Equation (11) and (12) set the initial backlogging and inventory levels. Equation (13) prevents a mold setup if a machine is not operated. Equation (14) is a symmetry breaking constraint. Equation (15) and (16) ensure a shift is not active if a previous shift is not active either. Overtime is prevented if a shift is not activated by Equation (17) and (18). Equation (19) secures that total available workforce is not exceeded. Equation (20), (21) and (22) ensure that the total available workforce is not exceeded for the three shifts. Equation (23) limits total realized overtime. Equation (24) mold usage does not exceed available mold quantities. Equation (25) prevents the production of a product if a mold capable to produce that product is not setup.

3. DECOMPOSITION HEURISTIC

It is shown in Bitran (1982) that the general CLSD problem is NP hard. Due to the NP hard nature of the CLSD problem, a decomposition strategy is developed in order to solve large instances. In the setting we consider, the major setups are related with the mold changes and require more time than the setups made on the mold version inserts (minor setups). Moreover, the major setup times are not sequence dependent. This gives an opportunity to decompose the model according to the setup types. Our proposed algorithm decomposes the problem into the major setups and minor setups and solves two hierarchical mathematical models.

In order to decompose the problem in setup types, all the products (versions), which the molds are capable to produce, are consolidated and referred as product types. First stage of the heuristic considers the major setups and solves lot sizing and scheduling problem without the sequence dependent setups on the products. The solution of the first stage provides the mold-machine allocation information. The information obtained in this stage is used to construct the set of sequence dependent setup decision variables, which are used in the second stage of the heuristic. The algorithm reduces the solution space of the problem dramatically as the sequence dependent setup variables for the molds which are not used on the machines are eliminated from the mathematical model. Additionally, defining the mold allocations in the first stage of the heuristic yields the removal of (25) from the model, which is a hard constraint connecting two binary decision variable sets, namely w_{ilt}^{I} and w_{mlt}^{M}.

The mathematical model for the first stage is presented below:

$$[MIN]\ Z = \sum_{m \epsilon S^M} \sum_{t \epsilon S^t} C_m^{HM} s_{mt}^M + \sum_{m \epsilon S^M} \sum_{t \epsilon S^t} C_m^{BM} b_{mt}^M + \sum_{l \epsilon S^l} \sum_{m \epsilon S^M} \sum_{t \epsilon S^t} C_m^{PM} q_{mlt}^M + \sum_{i \epsilon S^i} \sum_{j \epsilon S^i} \sum_{l \epsilon S^l} \sum_{t \epsilon S^t} C^{ST} T_m^{STM} w_{mlt}^M + C^{LAB}(\lambda^I + \lambda^{II} + \lambda^{III}) + \sum_{l \epsilon S^l} \sum_{t \epsilon S^t} C^{OT} \beta_{lt} \tag{26}$$

$$D_{mt} + s_{mt}^M + b_{mt-1}^M = s_{mt-1}^M + b_{mt}^M + \sum_{l \epsilon S^L} q_{mlt}^M \quad (\forall m \in S^M; \forall t \in S^T) \tag{27}$$

$$M w_{mlt}^M \geq q_{mlt}^M \quad (\forall m \in S^M; \forall l \in S^L; \forall t \in S^T) \tag{28}$$

$$\sum_{m \epsilon S^M} T_m^{CYCM} q_{mlt}^M + \sum_{m \epsilon S^M} T_m^{STM} w_{mlt}^M \leq T^C(\mu_{lt}^I + \mu_{lt}^{II} + \mu_{lt}^{III} + \Gamma^T \beta_{lt}) - \sum_{m \epsilon S^M} D_{it}^W \Omega_{mi} w_{mlt}^M \quad (\forall l \in S^L; \forall t \in S^T) \tag{29}$$

$$b_{mt}^M = B_m^0 \quad (\forall m \in S^M; t = 0) \tag{30}$$

$$s_{mt}^M = S_m^0 \quad (\forall m \in S^M; t = 0) \tag{31}$$

Equations (13)-(24)

$$w_{mlt}^M, \mu_{lt}^I, \mu_{lt}^{II}, \mu_{lt}^{III}, \beta_{lt} \in \{0,1\}$$

$$q_{mlt}^M, s_{mt}^M, b_{mt}^M \in \mathbb{R}^+$$

$$\lambda^I, \lambda^{II}, \lambda^{III} \in \mathbb{Z}^+$$

The objective function (26) minimizes the production, backlogging, inventory holding, setup, labor and overtime costs. Contrarily to the production cost in the exact model, Equation (26) takes the total number of products that the molds are producing. Equation (27) is the inventory balance equation. Equation (28) assures that if a mold is not mounted on a machine, then it is not allowed to produce products requiring that mold. Equation (29) is the capacity constraints which includes the expected minor setup times. Equation (4) in the exact model consider the exact sequence dependent setup times between product changes whereas Equation (29) separates major and minor setup times. The major setup times are included in the model as the total number of mold setups in the machines. The expected minor setup times are calculated as the total time of minor setups on a mold (i.e. total setup times of products produced on the mold) that can be made on each machine. Equations (30) and (31) are the initial backlogging and inventory levels. Equations (13) – (24) are same with the exact model.

The mold setup information taken from the first stage is used to generate sequence dependent setup variables in order to reduce the problem size in the second stage. All the decision variables related to versions (products) of a mold (product type) is added to variable pool of the second stage of the heuristic. The procedure used to generate the setup variables are shown in below algorithm:

Algorithm:

Solve first stage problem
Get solution values for w_{mlt}^M
for each w_{mlt}^M
  if $w_{mlt}^M = 1$ **then**
    for each product *i,j* in mold *m* **do**
      Add quadruple $< i, j, l, t >$ to the set $\bar{Z}$
    Next
  end if
next
Construct CLSD with $\bar{Z}$ and solve

$\bar{Z}$ is the set that contains $<$ i, j, l, t $>$ quadruples which are used to construct CLSD model instance in the second stage. The second stage mathematical model is presented below:

$$[MIN]\ Z = \sum_{i \epsilon S^i} \sum_{t \epsilon S^t} C_i^H s_{it}^I + \sum_{i \epsilon S^i} \sum_{t \epsilon S^t} C_i^B b_{it}^I + \sum_{l \epsilon S^l} \sum_{i \epsilon S^i} \sum_{t \epsilon S^t} C_i^P s_{it}^I + \sum_{i \epsilon S^i} \sum_{j \epsilon S^i} \sum_{l \epsilon S^l} \sum_{t \epsilon S^t} C^{ST} T_{ij}^{ST} \bar{z}_{ijlt} + C^{LAB}(\lambda^I + \lambda^{II} + \lambda^{III}) + \sum_{l \epsilon S^l} \sum_{t \epsilon S^t} C^{OT} \beta_{lt} \tag{32}$$

$$\sum_{i \epsilon S^I} T_i^{CYC} q_{ilt} + \sum_{i \epsilon S^I} \sum_{j \epsilon S^I} T_{ij}^{ST} \bar{z}_{ijlt} \leq T^C(\mu_{lt}^I + \mu_{lt}^{II} + \mu_{lt}^{III} + \Gamma^T \beta_{lt}) \quad (\forall l \in S^L; \forall t \in S^T; \forall < i, j, l, t > \in \bar{Z}) \tag{33}$$

$$\sum_{x \epsilon S^I \cup \{0\}} \bar{z}_{ixlt} = \sum_{y \epsilon S^I \cup \{0\}} \bar{z}_{yilt} \quad (\forall < i, x, l, t > \in \bar{Z};\ \forall < y, \iota, l, t > \in \bar{Z}) \tag{34}$$

$$\sum_{j \epsilon S^I \cup \{0\}} \bar{z}_{ijlt} \leq w_{ilt}^I \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T; \forall < i, j, l, t > \in \bar{Z}) \tag{35}$$

$$\sum_{i \epsilon S^I \cup \{0\}} \bar{z}_{i0lt} \leq \mu_{lt} \quad (\forall l \in S^L; \forall t \in S^T; \forall < i, 0, l, t > \in \bar{Z}) \tag{36}$$

$$\sum_{j \in S^I \cup \{0\}} \bar{z}_{0jlt} \leq \mu_{lt} \quad (\forall l \in S^L; \forall t \in S^T; \forall < 0, j, l, t > \in \bar{Z}) \tag{37}$$

$$u_{ilt} \leq |S^I| \quad (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{38}$$

$$u_{ilt} - u_{jlt} + |S^I| \bar{z}_{ijlt} \leq |S^I| - 1 \quad (\forall i \in S^I; \forall j \in S^I | i \neq j; \forall l \in S^L; \forall t \in S^T) \tag{39}$$

$$w_{ilt}^I \leq \mu_{lt}^I \; (\forall i \in S^I; \forall l \in S^L; \forall t \in S^T) \tag{40}$$

Equations (2)-(3)

Equations (11)-(23)

$$\bar{z}_{ijlt}, w_{ilt}^I, \mu_{lt}^I, \mu_{lt}^{II}, \mu_{lt}^{III}, \beta_{lt} \in \{0,1\}$$

$$q_{ilt}^I, s_{it}^I, b_{it}^I \in \mathbb{R}^+$$

$$u_{ilt}, \lambda^I, \lambda^{II}, \lambda^{III} \in \mathbb{Z}^+$$

Equation (32) is the objective function including the production, backlogging, and the inventory holding costs. Equation (33) is the inventory balance equation. Equation (34) - (39) refer to Equations (5) - (10) in the exact model but use limited sequence dependent setup variables eliminated in the first stage of the heuristic. Equation (40) prevents the production of product *i* if a setup decision is not made. Equations (2) - (3) and Equations (11) - (23) are used as they were used in the exact model.

4. NUMERICAL EXPERIMENTS

We use modified real-world instances obtained from Vestel Electronic plastic injection plant. The notation for the problem sets describe the number of products (I), number of molds (M), number of machines (L) and the number of periods (T). For instance, problem instance named I3/M2/L1/T7 has one machine with two molds, producing three products and problem considers seven production periods. Table 1 lists all the problem instances, which are used to compare the performances of the exact mathematical formulation and the developed decomposition heuristic. The total execution time is limited to 900 seconds for both the exact model and the heuristic approach. The objective function values of the best results obtained in the allotted time reported for both approaches. Table 1 presents two sets of information for each instance-approach combination: objective function value and resulting percentage gap, that is the percentage proximity of the attained solution to the lower bound obtained at the end of execution of the exact model. Mathematical models for both decomposition heuristic and exact methods are solved by the commercial solver CPLEX 12.6.

The results in Table 1 indicate that for small size instances (I3/M2/L1/T7 and I4/M2/L1/T7) both methods converge to proven optimal solutions. For these instances, both heuristic and exact model find the optimal solution in less than five seconds. For medium size instances I14/M6/L1/T7 and I20/M6/L1/T7, the heuristic method converges to local optimum solution, hence reported objective function value for the heuristic is larger than that of the exact model. For instances I14/M6/L1/T7 and I20/M6/L1/T7, the exact model finds the optimal solutions in the allotted time. For the other four medium size instances (I15/M7/L2/T7, I20/M14/L9/T7, I16/M10/L10/T7, I27/M7/L9/T7 and I28/M8/L13/T7) both approaches hit the time limit before convergence and it is seen that the objective function value of the solution found by the decomposition approach is better than that of the exact model. For these four instances, both solution approach hit the 900 second time limit and the best results obtained during the allotted time interval is reported. As the best solution found by the heuristic approach is superior to that of exact model, it could be inferred that the decomposition heuristic could be used for the larger size problem instances. To test this hypothesis, we create four large instances (I32/M12/L13/T7, I49/M26/L13/T7, I93/M71/L9/T7 and I92/M59/L9/T7) and run both algorithms using these instances. It is seen that the exact model cannot find a feasible solution in 900 second time limit, whereas the proposed heuristic is capable of providing feasible solutions for all of the four instances. As an extra step, we increase the allocated time 3600 seconds and run the exact model using CPLEX solver over these four instances in an IBM cloud machine with 60GB RAM. All four executions are terminated before hitting the 3600 seconds time limit with out of memory error and solver could not report any feasible solutions. This indicates that proposed heuristic is valuable for practical purposes as it can provide feasible solution to large size instances.

Table 1. Comparison of performance for exact and heuristic methods.

INSTANCE NAME	OBJ. VALUE		GAP	
	HEUR.	EXACT	HEUR.	EXACT
I3/M2/L1/T7	181519	181519	0,00%	0,00%
I4/M2/L1/T7	288110	288110	0,00%	0,00%
I14/M6/L1/T7	279150	277385	0,64%	0,00%
I20/M6/L1/T7	764113	763335	0,10%	0,00%
I15/M7/L2/T7	97147	97714	9,37%	10,01%
I20/M14/L9/T7	891302	904328	14,34%	16,01%
I16/M10/L10/T7	935845	950677	5,06%	6,72%
I27/M7/L9/T7	301901	324096	24,82%	33,99%
I28/M8/L13/T7	1318601	1434365	1,66%	10,58%
I32/M12/L13/T7	548141	*	-	*
I49/M26/L13/T7	566644	*	-	*
I93/M71/L9/T7	1124081	*	-	*
I92/M59/L9/T7	484893	*	-	*

5. CONCLUSIONS

We have demonstrated a decomposition heuristic for solving CLSD with secondary resources and workforce planning. The results show that the method is promising for finding feasible solutions even for large instances, where exact model cannot be solved in the limited time using one of the state-of-the-art commercial MIP solvers, CPLEX.

We are currently working on a column generation approach on the sequence dependent setup decision variables and elimination of the associated sub-tour elimination constraints. In the ongoing work, we are also modifying the way we connect consecutive periods in order to represents the real-world case better in modelling carry over setups between consecutive periods. In this approach we are aiming to solve very large instances of the complete problem for the injection molding plant (i.e. 87 machines 200+ molds and 300+ products) within less than an hour.

REFERENCES

Bitran, G. R. and Yanasse, H. H. (1982). Computational complexity of the capacitated lot size problem. *Management Science*, 28(10), 1174-1186.

Copil, K., Martin W., Herbert M. and Horst T. (2016). Simultaneous Lot sizing and Scheduling Problems: A Classification and Review of Models. *OR Spectrum*, 39(1), 1–64.

Haase, K. (1996). Capacitated lot-sizing with sequence dependent setup costs. *OR Spectrum*, 18(1), 51–59.

Hemig, C., Rieck, J. and Zimmermann, J. (2014). Integrated production and staff planning for heterogeneous, parallel assembly lines: An application in the automotive industry. *International Journal of Production Research*, 52(13), 3966–3985.

Kwak, I. and Jeong, I. (2011). A hierarchical approach for the capacitated lot-sizing and scheduling problem with a special structure of sequence-dependent setups. *International Journal of Production Research*, 49(24), 7425–7439.

Meyr, H. and Mann, M. (2013). A decomposition approach for the General Lot-sizing and Scheduling Problem for parallel production lines. *European Journal of Operational Research*, 229(3), 718–731.

Miller C. E., Tucker A. W. and Zemlin R. A. (1960), Integer programming formulations and traveling salesman problems, *J. ACM*, 7, 326–329.

Tavaghof-Gigloo, D., Minner, S. and Silbermayr, L. (2016). Mixed integer linear programming formulation for flexibility instruments in capacity planning problems. *Computers & Industrial Engineering*, 97, 101–110.