Modal characterization of additively manufactured TPMS structures: comparison between different modeling methods

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Abstract

The use of lattice structures has received increasing interest in various engineering applications owing to their high strength to weight ratio. Advances in additive manufacturing technologies enabled the manufacturing of highly complex lattice structures such as triply periodic minimal surface (TPMS) models in recent years. The application of simulation tools is expected to enhance the performance of these designs further. Therefore, it is vital to understand their accuracy and computational efficiency. In this paper, modal characterization of additively manufactured TPMS structures is studied using five different modeling methods for a beam, which is composed of primitive, diamond, IWP, and gyroid unit cells. These methods include (1) shell modeling, (2) solid modeling, (3) homogenization, (4) super-element modeling, and (5) voxelization. The modal characterization is performed by using modal analysis, and the aforementioned models are compared in terms of their computational efficiency and accuracy. The results are experimentally validated by performing an experimental modal testing on a test specimen, made of HS188, and manufactured by direct metal laser melting. Finally, the relationship between the modal characteristics and volume fraction is derived by carrying out a parametric study for all types of TPMS structures considered in this paper. The complex modal characteristics of different TPMS types suggest that they can be jointly used to meet the ever-challenging design requirements using the modeling guidelines proposed in this study.

Keywords Additive manufacturing · Sandwich structure · Triply periodic minimal surfaces · Finite element analysis · Modal analysis

1 Introduction

Cellular structures have been utilized in a wide range of engineering applications in recent years. Examples of these structures are also found in nature such as wood, honeycombs, bone, coral, and sponge [1]. These repeating lattice geometries provide many opportunities to develop structures with a lighter weight compared with those manufactured from conventional methods [2–4]. Faces and edges of cellular structures play a vital role in their weight, thermal insulation, energy absorption, heat transfer, and weight [5–8]. Cellular structures are classified as periodic

and non-periodic structures. It was demonstrated that the mechanical properties improve with the use of an increased number of narrow cells rather than lower number of wide cells while the volume fraction and part mass is kept constant [9].

Triply periodic minimal surfaces (TPMS) are well-known periodic cellular structures with distinctive characteristics. These cellular structures enable the manufacturing of composite structures that have superior mechanical properties [10]. Energy absorption capability is a desirable attribute of TPMS composites. According to Maskery [11], the stiffness characteristics of lattice structures depend on the geometry of the TPMS structure. The most common types of TPMS structures are gyroid, diamond, IWP, F-RD, neovius, and primitive. Moreover, the relation between cell type, volume fraction, and material plays an important role in the component design. There are experimental and theoretical studies for the characterization of these structures in the literature [12–15]. It is reported that the modulus and strength of the strut-based TPMS lattice structures, including gyroid and diamond types, extracted from compressive

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tests show good agreement with values calculated based on the Gibson-Ashby model. There have been numerous studies in the literature that compare TPMS geometries with face-centered cubic (FCC) and body-centered cubic (BCC) unit cells. It was shown that one of the well-known TPMS types, namely the double gyroid (DG) structure, contributes to higher stiffness and lower von-Mises stress values compared with FCC- and BCC-type cellular structures [16]. In one of these studies, Yanez et al. [17] have used a gyroid structure with porous titanium biomaterial on bone design. Experiments concluded that the stiffness of an ordinary gyroid as a result of multi-directional loading is more complex than pure compression loading. In another study, Maskery et al. [18] investigated the energy absorption characteristics of gyroid. The results show that DGs have nearly 3 times more energy absorption capability than BCC lattice for 50% compressive strain. Abueidda et al. [19] studied the mechanical properties of three different TPMS-based cellular structures experimentally and computationally using finite element modeling. The results show that Neovius and IWP types exhibit similar mechanical performance such as higher stiffness and strength values than the primitive structure. In another study, surface-based TPMS microlattices were compared with well-known strut-based microlattices in terms of their mechanical properties such as the elastic modulus, strength, and energy absorption characteristics [20]. The study concluded that the surface-based TPMS microlattices have superior mechanical properties compared with all strut-based microlattices. Two types of TPMS structures were examined in this study, and it was found out that the diamond structure has a higher effective elastic modulus than the gyroid cellular structure. In a more in-depth study on surface-based TPMS, six different TPMS foams were analyzed using finite element models to predict their effective electrical/thermal conductivities and elastic moduli [21]. The study concludes that the CLP-foam has the highest elastic modulus in the loading direction, followed by the IWP, neovius, gyroid, and primitive structures, respectively.

The mechanical properties such as compressive, tensile, and energy absorption characteristics of different types of TPMS were investigated experimentally and numerically in the literature. However, only a limited number of studies have focused on the modal characterization of TPMS-based cellular structures. Modal responses of various TPMS structures with different material and relative densities were studied by Altintas [22] in a wide range. The results show that the natural frequencies are sensitive to relative density and TPMS geometry. The highest natural frequencies are obtained by F-RD while the primitive structure has the lowest natural frequencies with the same material and relative density. In one of the recent studies on cellular structures, Elmadhi et al. [23] demonstrated that vibration

bandgaps may be controlled through the choice of cell type and volume fraction (VF) of the TPMS structures. Furthermore, Abueidda et al. [24] reported the acoustic band gaps, sound attenuation, and stiffness properties of different TPMS cellular structures.

Our previous work suggests that the gyroid structures have some unique vibration characteristics [25]. Moreover, it is concluded that surface mesh models can be efficient alternatives for conventional solid mesh models for modeling gyroid cellular structures. However, a comprehensive guideline for modeling TPMS structures is missing in the literature. There are several different finite element modeling approaches in the literature. For instance, linear, reduced-integration hexahedral elements were used, and a mesh convergence study was carried out by Maskery et al. [26]. Moreover, unit cells were voxelized and surfaces were patched into a bitwise three-dimensional arrays instead of relying on explicit modeling with traditional solid mesh algorithms. Each voxel array resembles a hexahedral element and this approach allows the modeling of a unit cell with identical elements [16]. However, modeling of surface-based TPMS unit cells with solid mesh and voxelization result in a large number of elements and it can be computationally expensive. An alternative method was presented by Simsek et al. [25] in order to improve the computational efficiency of the models without sacrificing accuracy. For that purpose, the representative volume element (RVE) method was utilized, in which material properties were extracted from the compression test for modeling of dynamic problems. However, the accuracy of the modeling method strongly depends on the size of RVE, so appropriate size determination can be a critical step in this approach. Even though the use of super-element modeling is common in the literature, the application of the method for modeling of TPMS cellular structures was not found. The reduction of computational effort can be achieved by using super elements which are suitable for periodic structures and linear problems [27].

The aim of this study is to gain more insight on the modal characteristics of TPMS structures with different relative densities using different modeling approaches. More specifically, the computational efficiency and accuracy of the five modeling methods are compared to determine the modal characteristics: (1) shell models, (2) solid models, (3) homogenization, (4) super-element models, and (5) voxelization. The comparisons on the accuracy are made by calculating the relative error between the simulation results and experimental modal testing. The efficiency of these modeling approaches is compared on (i) modeling effort, (ii) simulation run time in terms of CPU time, and (iii) data management in terms of the size of the models and simulation results. The methods are demonstrated on a beam sandwich structure, which is based on four different TPMS

unit cells, including primitive, diamond, IWP, and gyroid, manufactured from commercially available alloy HS188 utilizing direct metal laser melting (DMLM) method. Besides, the effects of the relative density on the modal characteristics of these structures are studied in detail.

The remainder of the paper is organized as follows: the geometry of the lattice structure is presented in Section 2. Then, the five different modeling approaches are detailed in Section 3. Thereafter, computational efficiency and accuracy of the methods are compared in Section 4. The modal characterization of different geometries is explained in Section 5. Finally, conclusions and future work are given in Section 6.

2 Lattice structure geometry

Triply periodic minimal surfaces (TPMS) have a mean curvature of zero at every point on the surface. Four lattice types of TPMS structures were considered in this study: primitive (P), diamond (D), Schoen IWP, and gyroid (G). These lattice structures can be generated with arbitrary numbers of cells and volume fractions (VF) by finding the $U=0$ iso-surfaces and treating these surfaces as the boundary between solid and void material phases. TPMS structures can be mathematically expressed using implicit methods in the following forms:

$$f_{\text{Primitive}}(x, y, z) = (\cos(k_x x) + \cos(k_y y) + \cos(k_z z))^2 - t^2 \quad (1)$$

$$f_{\text{Diamond}}(x, y, z) = (\sin(k_x x) \sin(k_y y) \sin(k_z z) + \sin(k_x x) \cos(k_y y) \cos(k_z z) + \cos(k_x x) \sin(k_y y) \cos(k_z z) + \cos(k_x x) \cos(k_y y) \sin(k_z z)) - t^2 \quad (2)$$

$$f_{\text{IWP}}(x, y, z) = 4[\cos(k_x x) \cos(k_y y) + \cos(k_y y) \cos(k_z z) + \cos(k_z z) \cos(k_x x) - (\cos(2k_x x) + \cos(2k_y y) + \cos(2k_z z) + \cos(2k_x x) \cos(2k_y y) \cos(2k_z z))] - t^2 \quad (3)$$

$$f_{\text{Gyroid}}(x, y, z) = (\cos(k_x x) \sin(k_y y) + \cos(k_y y) \sin(k_z z) + \cos(k_z z) \sin(k_x x))^2 - t^2 \quad (4)$$

When creating TPMS geometries, it is important to represent the geometry of TPMS-based structures accurately. In the current study, MATLAB software is used for constructing the 3D geometry of the TPMS structure by using the appropriate formula given in Eqs. 1, 2, 3, and 4. Besides, MATLAB includes a capability of exporting a

stereo-lithography (STL) data of the corresponding TPMS structures.

In Eqs. 1, 2, 3, and 4, t can be used as a variable to control the volume fraction of the resulting cellular structure. There is a unique relationship between the parameter t and relative density ρ^* for each TPMS cell type, which is shown in Fig. 1. These TPMS geometries with different volume fractions are presented in Fig. 2.

3 Finite element modeling methods

Five different modeling methods were used for the representation of aforementioned types of TPMS structures. To the author's best knowledge, super-element modeling has not been previously applied to these structures in the literature. Modeling methods are explained in Sections 3.1 to 3.5.

3.1 Shell models

The shell modeling methodology is shown in Fig. 3a. Even though the method is shown on gyroid structures, it is applicable to other TPMS structures. U field of the TPMS equation is generated as the first step. Then, the midsurface is extracted at $t = 0$, and Delaunay triangulation at $U = 0$ is applied. The resulting geometry is then meshed with shell elements. For this purpose, Shell181 element type is used in ANSYS software. This element is most suitable for analyzing thin to moderately thick shell structures [28]. The finite element model has 4 nodes, with each node having six degrees of freedom (DOFs). These DOFs are the x, y, z translations and rotations. It is one of the most commonly used finite element models in the literature as it is suited for

Fig. 1 Relationship between the TPMS equation parameter t and relative density ρ^*

Fig. 2 TPMS structures with different volume fraction (a) 0.2 VF, (b) 0.4 VF, (c) 0.6 VF, and (d) 0.8VF

linear and nonlinear applications. One of the most critical properties for the shell element is the thickness. Therefore, the thickness of the walls is varied until the correct mass is achieved for a given volume fraction. While this modeling approach is considered as simple and fast in terms of pre-processing, one important limitation is related to the uniform representation of the TPMS walls. This assumption may not hold especially at the junction regions.

3.2 Solid models

A similar approach as shell modeling is taken as shown in Fig. 3b. The first step is common for both methods. However, after the extraction of the midsurface and application of the Delaunay triangulation, the resulting geometry is modeled using three-dimensional solid elements. Specifically, Solid45 element in ANSYS software is used for modeling each TPMS structure. Solid45 finite element is defined by eight nodes having three degrees of freedom at each node, which are x, y, and z translations of each node.

This type of element takes into account the stress stiffening effects and large deflections. One important modeling consideration is the number of elements throughout the gyroid wall thickness as using lesser number of elements would result in shear locking phenomenon [28]. In this study, the wall thickness of the TPMS structure is modeled with minimum four elements and gradually increased up to eight elements as a function of volume fraction.

3.3 Homogenization

Another modeling method considered in this research is homogenization. The essence of this method lies in the modeling overall lattice envelope with solid elements instead of representing lattice topology explicitly, thereby providing great convenience. However, in this approach, constitutive material properties cannot be used to simulate TPMS structures due to variation in the shape. Therefore, material properties of TPMS structures should be extracted experimentally or computationally. In the present study, the

Fig. 3 Different modeling approaches for gyroid lattice structures (a) Shell modeling, and (b) solid modeling

finite element (FE) method is utilized in order to extract the effective elastic properties. For this purpose, the TPMS walls were modeled using linear hexahedral elements. An element aspect ratio close to unity is enforced in order to minimize FEA errors associated with distorted elements. A displacement-controlled loading in z-direction, equivalent to 1% strain, is applied to the nodes of the top plane of

the lattice model. The nodes of the bottom plane are fixed in z-direction, while translations in x- and y-directions are not constrained. Periodic boundary conditions are applied to the nodes of all other free surfaces in the corresponding direction. The detailed representation of the loading and boundary conditions is provided in Fig. 4. The mesh size is determined by performing a mesh convergence study.

Fig. 4 Boundary conditions and loading

Once the displacements of the nodes of the top surface are calculated, the equivalent elastic modulus of the unit cell E^* is determined from Eq. 5:

$$E^* = \frac{F\ell}{A\Delta\ell} \quad (5)$$

where F is the total reaction force at the top plane nodes of the lattice structure, ℓ is the original height of the TPMS (12 mm), A is the cross-sectional area of the TPMS projection area (144 mm²), and $\Delta\ell$ is the displacement of the top plane nodes in the loading direction (0.12 mm). Once the equivalent elastic modulus is determined for a specified relative density, the analysis is repeated for different relative densities ranging from 0.1 to 0.8. Then, an empirical relation between the equivalent elastic modulus and relative density is derived using the Gibson-Ashby model in Eqs. 6 and 7:

$$\rho^* = \frac{\rho}{\rho_c} \quad (6)$$

$$\frac{E^*}{E_c} = C(\rho^*)^n \quad (7)$$

where E^* and ρ represent the elastic modulus and density of lattice structure, respectively. E_c and ρ_c are the elastic modulus and density of the substrate material, respectively. C and n are constant numbers for each structure, and ρ^* is the relative density.

Figure 5 shows the variation of the elastic modulus of the TPMS unit cells with the change of the relative density between 0.1 and 0.8. It is remarkable that elastic modulus considerably varies from one TPMS unit cell to another. The highest elastic modulus at 0.1 relative density is attained by the primitive lattice in the z-direction, followed by diamond, IWP, and gyroid. It is clearly seen from Fig. 5 that the order of TPMS lattices with respect to elastic modulus changes at higher relative densities.

The Gibson-Ashby's prediction of relative elastic modulus for primitive, diamond, IWP, and gyroid structures is in good agreement with analysis results at relative densities less than 0.5. However, the correlation is low for higher

Fig. 5 Homogenization results for $1 \times 1 \times 1$ unit. TPMS structures: (a) primitive, (b) diamond, (c) IWP, and (d) gyroid

relative densities. Therefore, a polynomial function was used to fit a curve for the relative density values greater than 0.5. The Gibson-Ashby and polynomial equations are given in Eqs. 8 and 9, respectively.

$$\begin{aligned} f_{Primitive} &= 0.97\rho^*^{1.54} \\ f_{Diamond} &= 0.85\rho^*^{1.43} \\ f_{IWP} &= 0.99\rho^*^{1.53} \\ f_{Gyroid} &= 0.72\rho^*^{1.65} \end{aligned} \quad (8)$$

$$\begin{aligned} f_{Primitive} &= -22.86\rho^*^5 + 40.24\rho^*^4 \\ &\quad - 23.04\rho^*^3 + 5.78\rho^*^2 - 0.06\rho^* \\ f_{Diamond} &= 0.67\rho^*^3 + 0.14\rho^*^2 + 0.39\rho^* \\ f_{IWP} &= 0.39\rho^*^3 + 0.55\rho^*^2 + 0.32\rho^* \\ f_{Gyroid} &= 3.76\rho^*^4 - 4.12\rho^*^3 + 0.16\rho^*^2 - 0.16\rho^* \end{aligned} \quad (9)$$

The static analysis is performed for a single unit cell with aforementioned periodic boundary conditions. Additionally, a convergence study is performed to determine the magnitude of the representative volume element (RVE) that gives the accurate macroscopic values in terms of equivalent elastic modulus. Accordingly, TPMS structures with cell tessellations of volume fraction of 0.2, ranging from $1 \times 1 \times 1$ to $5 \times 5 \times 5$, are generated, as shown in Fig. 6.

Figure 7 provides the trends in variation of elastic modulus by the number of lattice cells. It can be seen that

lattice structures with fewer cells have the lowest elastic moduli. Afterwards, an exponential function was fit to the data as given in Eq. 10:

$$E^* = a \exp(-bm) + E_\infty \quad (10)$$

where m is the lattice order, and a and b are constant parameters. E_∞ is the upper bound on the relative elastic modulus representing the infinitely long lattice structure. For the primitive, diamond, IWP, and gyroid lattice structures with relative density of 0.2, subjected to compressive load, E_∞ was calculated as 9.0, 19.1, 17.0, and 13.9 GPa, respectively. Figure 7 shows that the relative elastic modulus of the all lattices converges at $5 \times 5 \times 5$ cell size with an error less than 1% except for primitive.

Simulation results indicate that for the purpose of elastic modulus determination there is no benefit of modeling cubic lattice structures with greater than $5 \times 5 \times 5$ cells. It can be also be concluded from results that a single unit cell gyroid lattice model without periodic boundary conditions may underestimate the elastic modulus with an error of 41%. On the other hand, a single unit cell TPMS periodic boundary condition can predict elastic modulus with just 10% error for all TPMS types except for the primitive

Fig. 6 $1 \times 1 \times 1$ to $5 \times 5 \times 5$ gyroid unit cells with relative density, $\rho^* = 0.2$

Fig. 7 Homogenization results for TPMS structures: (a) primitive, (b) diamond, (c) IWP, and (d) gyroid

structure. Interestingly, a single primitive unit cell with periodic boundary condition estimates the effective elastic modulus as twice as higher than the value obtained from Fig. 7. This indicates that elastic modulus characterization with different lattice orders helps in finding the effective elastic modulus accurately. However, it is not sufficient to capture the elastic behavior of different unit cell types. This could be explained by observing their stretching- or bending-dominated characteristics. It was shown earlier that primitive cells obey stretching-dominated behavior while diamond and gyroids display bending-dominated behavior [11, 29]. They concluded that pure stretching deformation is followed by structural buckling for primitive cell and effective elastic modulus is higher than others. The findings from present study are in good agreement with the literature. Additionally, reaction forces on periodic boundary planes are extracted from single unit cell models as shown in Fig. 4 to validate deformation characteristics. Results show that overall reaction force on periodic boundary plane of

primitive unit cell is three times higher than the others. The results also suggest that the primitive cell benefits from periodic boundary conditions owing to its stretching dominant behavior. Hence, determination of appropriate effective elastic modulus for the structures filled with the primitive unit cells is quite challenging for most cases. In this study, the effective elastic modulus was calculated using a single unit cell analysis with periodic boundary conditions and used for the subsequent analysis of primitive unit cells sandwiched between two plates.

From a more general perspective, relative elastic modulus characterization with larger numbers of cells is an appropriate approach for continuous structures infilled with TPMS. With larger numbers of cells in the lattice, smaller proportions of them are located at the boundaries, and therefore their contribution to the stiffness of the whole structure is reduced. This results in homogeneous structural deformation and stress distribution, thus providing a more homogeneous porous solid.

Once the coefficients of the Gibson-Ashby formula are extracted from the results of computational study, relative elastic modulus of TPMS can be obtained for the structure corresponding to a specific relative density value.

3.4 Super-element models

TPMS-based structures consist of many unit cells. Therefore, the analysis of the full finite element model of such structures is computationally inefficient. The super element method is a numerical simplification of the number of degrees of freedom of the full finite element model to improve computational efficiency. The essence of the modal reduction is based on the condensation of the structure matrices on the selected master of DOFs [27]. There has been a vast body of literature on condensation methods. The most commonly used techniques are static and dynamic condensation. Guyan reduction method, as one of the most commonly static techniques, is applicable to static problems or low frequencies close to low eigenfrequencies. However, the accuracy degrades significantly for the higher frequencies. Therefore, the component mode synthesis (CMS) was preferred over the Guyan reduction method due to the dynamic nature of the problem in this study. The accuracy of the CMS method depends on the number of the eigenmodes retained in the modal solution. Therefore, an initial numerical study was performed to understand the relationship between the accuracy of the CMS compared with experimental data, to decide on how many modes should be retained in the solution. It was found out that a similar response is obtained by retaining the eigenmodes beyond 60 kHz. Therefore, the frequencies up to 60 kHz are considered in this study. The reader is referred to [30] for more detailed information on the advantages and disadvantages of the modal reduction methods. Once a super element library is generated with different unit cell sizes and relative densities using CMS, these super-element models can be integrated to analyze the dynamic characteristics of the full model.

3.5 Voxel mesh

Thus far, various approaches for effective modeling of cellular structures have been described. Voxel-based elements are also widely used to simulate the mechanical properties of lattice structures thanks to their ease of modeling and computational efficiency. Fundamentally, voxelization converts the geometry represented by surfaces into a bit-wise three-dimensional arrays instead of generating the mesh surfaces as in solid modeling. Then, this array can be directly mapped into the hexahedral mesh. The method is most suitable for more complex lattice structures, which can be realized using additive manufacturing, owing to the

reduction of computation time and geometrical flexibility by approximation. In this modeling approach, it is important to determine the number of voxels used to avoid any loss of geometric precision. Therefore, a convergence study is necessary by increasing the resolution of voxels and determining the mesh size. Although voxel representation brings flexibility in modeling of complex geometries, a large number of voxels may be required to represent those models which will tremendously slow down the computational speed.

4 Comparison between experimental and simulation results

The accuracy and efficiency of the aforementioned modeling methods are compared with each other in this section. The accuracy of the models is judged based on their prediction compared with experimental results from the modal tests. For this purpose, a complete $24 \times 25.6 \times 192$ mm beam was generated, consisting of $24 \times 24 \times 192$ mm TPMS structures with a unit cell size of 12 mm. The structure was sandwiched between upper and lower sheets, each with a thickness of 0.8 mm. Modal analyses were performed using ANSYS software, and natural frequencies in the frequency range of 10 Hz to 14 kHz are extracted. Five different models of the IWP structure are shown in Fig. 8. Selective laser melting (SLM) has been used to manufacture the lattice structures made of HS188. The density, Poisson's ratio, and elastic modulus of HS188 are 8.98 g/cm^3 , 0.3, and 232 GPa, respectively [31].

Manufacturing of the test specimen using SLM is detailed in Section 4.1. Experimental set-up and details of the modal testing for the verification of the numerical studies are presented in Section 4.2. Correlation results, advantages, and disadvantages of the modeling approaches are discussed in Section 4.3. The validity of the shell modeling approach is discussed in detail as one of the most common modeling approaches in Section 4.4. Finally, miscellaneous modeling considerations such as computational efficiency and post-processing file size are summarized in Section 4.5 as a general guideline for the user.

4.1 Manufacturing of test specimen using SLM

Additive manufacturing is a specific process, which enables the manufacturing of a three-dimensional component layer by layer by depositing material in contrast to traditional manufacturing, where the removal of material is the main process of finalizing the components. AM technology has been widely applied in many businesses owing to its flexibility in manufacturing complex parts and design features [32].

A variety of AM processes have been industrialized in the last two decades, with their known advantages and

Fig. 8 Finite element models for IWP structure: (a) solid, (b) shell, (c) RVE, (d) super element, (e) voxel mesh

limitations. Although these methods are different, they all use sliced geometric data, store layer information in CAD format, and manufacture the components layer by layer by depositing material using a suitable fusing technology. For metals, many modalities have been industrialized, and many more are still under development. It is well known in the

literature that the selection of the correct modality plays a critical role to benefit fully from the AM technologies and achieve desired design specifications [29].

SLM or DMLM is a powder bed fusion (PBF) process, which is one of the most widely researched AM technologies in recent years. The primary advantage of

Fig. 9 Test samples: (a) primitive, (b) diamond, (c) IWP, (d) gyroid

DMLM is to produce fully dense metal components with complex forms generated from computer-aided design tools [14]. From the manufacturing point of view, it has great potential to fabricate periodic structures with fine features at a high resolution that are impractical or impossible to be manufactured by conventional methods. In the open literature, SLM has been recently employed to build metal cellular structures using several different materials [15, 33, 34]. In the present study, sandwich structure test specimens made from various TPMS unit cell types were manufactured for modal testing; see Fig. 9.

4.2 Experimental verification via modal testing

The roving hammer method was used in the modal testing as a means of quantifying the accuracy of different models. The experimental set-up for the determination of the modes is shown in Fig. 10. The structure is excited at five locations numbered from 1 to 5 using a DJB IH-2 impact hammer. The impact hammer has a force range of 2000 N and 2.36 mV/N sensitivity. Accelerations are measured during modal testing from a stationary point. For this purpose, a three-directional accelerometer was used with a resolution of 100 mV/g (type: KS903.100). Dewesoft SIRIUS Data Acquisition system is used for data acquisition. Frequency response functions from the modal testing are post-processed to determine the modes and mode shapes. Each impact is repeated three times to ensure the repeatability of the results.

4.3 Results

The acceleration spectrum for point 1 (shown in Fig. 10) plotted from experimental modal testing data for primitive, diamond, IWP, and gyroid structures is shown in Fig. 11. Frequency response functions for these four structures are very similar for the frequencies up to 7100 Hz. However, a variation on the rest of the frequency spectrum is observed beyond this frequency limit. For example, the primitive structure has the highest 3rd bending mode corresponding to the value of 11,600 Hz, while the same mode shape for the diamond structure has the lowest value at 10,710 Hz.

Experimental and numerical results for the three bending modes are summarized for primitive, diamond, IWP, and

Fig. 10 Experimental set-up for modal testing

Fig. 11 Acceleration spectrum for point 1 from modal testing for primitive, diamond, IWP, and gyroid structures

gyroid structures in Tables 1, 2, 3, and 4, respectively. The relative error results between the experimental and simulations are also presented as a measure of accuracy.

The relative error between the experimental and simulation results ranges from 1.2 to 11% for the prediction of the first three bending modes. The minimum error is 1.2% using the super-element method for the second bending mode for the primitive structure. The maximum relative error of 11% is achieved using the super-element method for the first bending mode for the primitive structure.

Similar conclusions can be drawn for diamond, IWP, and gyroid structures. The relative error for these structures can, in general, be predicted with relative error less than 10% for these modes using shell-, solid-, and super-element methods. These results suggest that all these modeling approaches can adequately represent these structures in this frequency range.

Modeling of the TPMS surfaces using homogenization method yields the most accurate results compared with modal testing for the prediction of the first bending mode. The error between the modal testing and homogenization method for the first bending mode is 2.3%, 0.9%, 1.3%,

and 5.6% for the primitive, diamond, IWP, and gyroid structures, respectively. However, the relative error of the homogenization method is significantly higher for the second and third bending modes. For example, even though the relative error between the experimental and homogenization methods is only 0.9% for the first bending mode, the relative error is 12.6% and 19.4% for the second and third modes, respectively. These results indicate that the material properties derived using homogenization may be frequency dependent. The authors acknowledge this investigation as future work.

4.4 Validity of shell modeling

The results of the shell modeling of TPMS structures with a relative density of 0.2 indicate the high accuracy of this modeling approach. However, the accuracy of this modeling approach is expected to degrade for structures with increasing relative density. Therefore, in this section, the validity of the shell modeling approach is discussed. For that purpose, the results of the shell models were compared with those from voxel methods for TPMS structures with different relative densities. The comparison study is performed on beam models with $24 \times 24 \times 192$ mm without plates for four different unit cell types. The relative density is varied from 0.1 to 0.5 with 0.1 increments. The relative difference between results of voxel and shell models is presented for different mode shapes including bending, torsion, and longitudinal.

The relative error between shell and voxel models is calculated to quantify the accuracy of the shell modeling. The results for the 1st bending, 1st torsion and longitudinal modes are plotted in Figs. 12, 13, and 14, respectively. The results indicate that the relative error increases with increasing relative density for all the TPMS structures. The only exception occurs for some of the TPMS structures at a relative density of 0.2. When comparing shell- and voxel-type models, the minimum relative error for diamond structures with relative density of 0.2 is observed for the 1st bending mode. This result can be explained by the fact that a beam modeled with shell elements estimates

Table 1 Comparison between experimental and simulation results for primitive structure

Methodology	Calculated mass (kg)	Bending 1b (Hz)/rel. error (%)	Bending 2b (Hz)/rel. error (%)	Bending 3b (Hz)/rel. error (%)
Experimental	0.268	3319/-	7020/-	11,600/-
Shell	0.265	2978/10.3%	7141/1.7%	12,031/3.7%
Solid	0.265	3035/8.6%	7300/4.0%	12,337/6.4%
Homogenization	0.265	3244/2.3%	7611/8.4%	12,628/8.9%
Super element	0.265	2955/11.0%	7103/1.2%	11,990/3.4%
Voxel	0.265	3047/8.2%	7315/4.2%	12,365/6.6%

Table 2 Comparison between experimental and simulation results for diamond structure

Methodology	Calculated mass (kg)	Bending (Hz)/rel. (%)	1b error	Bending (Hz)/rel. (%)	2b error	Bending (Hz)/rel. (%)	3b error
Experimental	0.290	3300/-		6833/-		10,710/-	
Shell	0.265	3162/4.2%		7150/4.6%		11,530/7.7%	
Solid	0.265	3214/2.6%		7297/6.8%		11,801/10.2%	
Homogenization	0.265	3270/0.9%		7693/12.6%		12,793/19.4%	
Super element	0.265	3251/1.5%		7461/9.2%		12,161/13.5%	
Voxel	0.265	3174/3.8%		7188/5.2%		11,601/8.3%	

Table 3 Comparison between experimental and simulation results for IWP structure

Methodology	Calculated mass (kg)	Bending (Hz)/rel. (%)	1b error	Bending (Hz)/rel. (%)	2b error	Bending (Hz)/rel. (%)	3b error
Experimental	0.284	3300/-		6781/-		10,760/-	
Shell	0.265	3071/6.9%		6965/2.7%		11,253/4.6%	
Solid	0.265	3124/5.3%		7101/4.7%		11,495/6.8%	
Homogenization	0.265	3257/1.3%		7903/16.5%		12,713/18.2%	
Super element	0.265	3130/5.2%		7121/5.0%		11,537/7.2%	
Voxel	0.265	3109/5.8%		7094/4.6%		11,497/6.9%	

Table 4 Comparison between experimental and simulation results for gyroid structure

Methodology	Calculated mass (kg)	Bending (Hz)/rel. (%)	1b error	Bending (Hz)/rel. (%)	2b error	Bending (Hz)/rel. (%)	3b error
Experimental	0.284	3180/-		6880/-		11,040/-	
Shell	0.265	3050/4.1%		7072/2.8%		11,606/5.1%	
Solid	0.265	3115/2.0%		7267/5.6%		11,988/8.6%	
Homogenization	0.265	3003/5.6%		6794/1.3%		10,962/0.7%	
Super element	0.265	3121/1.9%		7289/5.9%		12,035/9.0%	
Voxel	0.265	3079/3.2%		7135/3.7%		11,713/6.1%

Fig. 12 Relative error (%) between voxel and shell models on the first bending mode **Fig. 13** Relative error (%) between voxel and shell models on the first torsion mode

Fig. 14 Relative error (%) between voxel and shell models on the longitudinal mode

natural frequencies with higher frequency values than a voxelized beam between 0.1 and 0.2 relative densities while the opposite is valid for the higher relative densities. Similar trends are observed for torsion and longitudinal modes. More specifically, shell modeling at a relative density of 0.2 is the most accurate on torsion mode for primitive and gyroid structures and on the longitudinal model for the gyroid structure.

Another important finding is that the results of both methods are in good agreement up to 0.2 relative density with a relative error of less than 5% for all TPMS unit cells. Furthermore, the maximum difference between the shell and voxel models occurs for diamond on bending mode, IWP on torsion mode, and primitive on longitudinal mode. The maximum relative error is more than 20% for bending and longitudinal modes and around 15% for the torsion mode for relative density of 0.5. These errors are considered unacceptable for many engineering applications.

As a rule of thumb, the present study suggests that shell models can be safely preferred for all unit cell types and mode shapes for relative densities below 0.2. Beyond this threshold, the use of shell elements mainly depends on the TPMS type, relative density, and mode shape as well. The weaknesses of shell modeling due to element formulation and geometry mismatching in the transition region become prominent for all TPMS unit cell types for increasing relative density.

4.5 Miscellaneous modeling considerations

The development of accurate and efficient modeling and analysis strategies to characterize the mechanical performance of complex lattice structures has a great potential to benefit fully from the AM technologies for a wide range of engineering applications. A comparative study between different modeling methods is presented in terms of their modeling effort, computation time, data management, and applicability in this study.

The shell meshing and voxel modeling were identified as the fastest methods in terms of the pre-processing of the CAD data, considering the modeling effort needed to prepare the models. Besides, shell modeling stood out among the other methods in terms of computation time, computer hardware requirements, and output data size. However, this strategy is only valid for lower element densities.

The homogenization-based modeling was reported to be the fastest strategy in terms of computation time owing to representative modeling instead of explicit geometry modeling. The overall element number can be reduced drastically, making the homogenization-based modeling the most attractive from the data management point of view as well. Even though the extraction of the elastic properties requires some additional modeling effort, it is still comparably less than the pre-processing time needed for solid- and super-element modeling. Even though this method is applicable for a wide range of relative densities, the modal testing results indicate that homogenization in one direction may not adequately capture the dynamic performance of TPMS-based cellular structures at higher frequencies. The accuracy of the calculated material property values in three directions could be further improved by taking their frequency dependency characteristics into account. The authors acknowledge this as a future study.

The findings from this study indicate that solid modeling using hexahedral and tetrahedral elements is computationally quite expensive, even though high-performance computing (HPC) systems are available. The main reason is that TPMS-based structures are thin wall structures, and therefore, a certain number of elements should be used throughout the thickness to improve accuracy. However, this increases the element numbers and the number of degrees of freedom of the model drastically. For instance, more than a million of solid elements were used to model a sandwich beam filled with IWP unit cells with a relative density of 0.2, compared with 150K and 50K elements for shell and homogenization models, respectively. The element number increases exponentially for TPMS structures with higher relative density. Besides, the output data size from the analysis is comparably larger than other methods, which can be considered as another drawback.

The super-element method is considered to overcome the aforementioned drawbacks of solid modeling for the first time in the open literature. The results indicate that the super-element method is almost twice as fast as solid modeling, while it is still slower than shell and homogenization methods. Once a unit cell is condensed as a super element, it can be used repeatedly in different FE models. From this modularity perspective, the super-element method is superior to solid modeling. Besides, the

size of the simulation data for the super element is almost 10% of that of solid modeling. This is very advantageous for the disc space, especially when these models are used in the optimization studies.

Though the voxel meshing strategy emerged out as one of the fastest and robust methods in terms of generating the model, it was only slightly faster than solid modeling from computational efficiency stand-point. Besides, the size of the simulation data from the analysis is comparable with that of solid modeling.

5 Modal characterization

In this section, the relationship between the modes of the TPMS structures and their volume fraction is analyzed. For

this purpose, the volume fraction is varied between 0.1 and 0.5 with an increment of 0.1. Modal analysis is performed for each TPMS structure corresponding to these volume fractions. The results from these analyses are plotted in Fig. 15 for bending, torsional, and longitudinal modes.

There is a linear relationship between the bending/longitudinal modes and the volume fraction for the primitive structure. The bending modes are the most sensitive to the volume fraction. More specifically, the first and second bending modes are increased by almost 2.5 times by increasing the volume fraction from 0.1 to 0.5. Torsion modes 1 and 2 are least affected by the volume fraction. They are increased only by 40% by increasing the volume fraction from 0.1 to 0.5.

The first bending mode of the diamond structure is not sensitive to the volume fraction. This suggests that it is

Fig. 15 Modal characterization of TPMS structures: (a) 1st bending mode, (b) 2nd bending mode, (c) 3rd bending mode, (d) 1st torsion mode, (e) 2nd torsion mode, and (f) longitudinal mode

Table 5 Comparison of different modeling approaches

Modeling strategy	Modeling ease	Computational efficiency	Data management	Applicability
Shell	••••	•••••	•••••	••
Solid	•	•	•	•••••
Homogenization (PBC)	••	••••	••••	••••
Super element	•••	•••	•••	•••••
Voxel	•••••	••	••	•••••

not possible to increase the bending mode significantly by increasing the volume fraction. Similarly, the sensitivity of the second bending and longitudinal modes to the relative density is quite low. For the other modes, a linear relationship is observed between the mode and volume fraction. This simple relation can be used to understand the volume fraction to increase the mode value to a desired target. The most sensitive mode to volume fraction is the second torsion mode. By increasing the volume fraction from 0.1 to 0.5, the second torsion mode is increased from 7302 to 10,563 Hz.

The bending modes of IWP lattice structures are more sensitive to the relative density compared with other modes. By increasing the relative density from 0.1 to 0.5, the bending modes are increased by 70%. However, only 40% of improvement is achieved with the increase of the relative density for the torsion modes.

There is a linear relationship between modes and volume fraction for the gyroid lattice structure. The volume fraction affects the bending modes the most. For example, almost 60% improvement is achieved by changing the volume fraction from 0.1 to 0.5. Even though the torsion modes and longitudinal modes are also altered by changing the volume fraction, it is less effective. These modes are only improved by 40% by increasing the volume fraction from 0.1 to 0.5. Comparison of the different types of TPMS structures yields the following conclusions:

- A similar trend is observed for all types of TPMS structures on the first three bending modes. Even though the diamond structure has the highest mode for the relative density of 0.1, the IWP structure has the highest mode for the relative density of 0.5. Diamond- and IWP-type cell structures exhibit the same bending mode for a relative density value of 0.3. The same conclusion can be drawn from the comparison of primitive and gyroid structures.
- The primitive structure provides the highest torsional characteristics in terms of torsion modes 1 and 2, followed by gyroid, diamond, and IWP structures. Diamond and gyroid structures exhibit very close performance on the torsion modes.
- A similar trend for the longitudinal mode is observed as the bending modes for the TPMS structures. The

diamond structure has the highest longitudinal mode for the relative density of 0.1. However, the IWP structure has the highest longitudinal mode for the relative density of 0.5.

6 Conclusions

The modal performance of different TPMS structures was characterized in this study. For this purpose, the accuracy and computational efficiency of different modeling approaches were compared. More specifically, primitive, diamond, IWP, and gyroid unit cells were considered using five different modeling approaches: (1) shell modeling, (2) solid modeling, (3) homogenization, (4) super-element modeling, and (5) using voxel elements. These models are assessed in terms of their modeling effort, computational efficiency, accuracy, and output size. Table 5 summarizes the comparison of these methods based on the detailed discussion from Section 4. This information is important as to which model should be used under which circumstances. To the author's best knowledge, super-element modeling was one of the first applications to model these structures in the literature. The accuracy of these models is verified against experimental data. For that purpose, a modal test on the aforementioned structures was performed on test specimen made of HS188 and manufactured by direct metal laser melting.

Modal characterization of all TPMS structures is derived with respect to the relative density. The complex relationship between the modes, TPMS types, and relative density results are discussed in detail. Such information is expected to enable more efficient and competitive structures using a combination of various TPMS structures.

Relative elastic moduli values for primitive, diamond, IWP, and gyroid cell type structures obtained using Gibson-Ashby's prediction model seem to display a reasonable accuracy for relative density values less than 0.5. Using a polynomial equation improves the accuracy of the fit significantly for relative density values greater than 0.5 for the primitive, diamond, IWP, and gyroid structures.

The relative error for these structures can be predicted with relative error less than 10% for these modes using shell-, solid-, and super-element methods. These results

suggest that all these modeling approaches can adequately represent the structures in this frequency range. The accuracy of the homogenization method is high for predicting the first bending of the TPMS surfaces. However, the accuracy degrades comparably for the higher modes. These results indicate that the material properties derived using homogenization may be frequency dependent. The authors acknowledge this investigation as future work.

Another important finding is that the results of both methods are in good agreement up to 0.2 relative density with a relative difference less than 5% for all TPMS unit cells. Beyond this threshold (0.2), the use of shell element mostly depends on TPMS type, relative density, and mode shape as well. Error sources inherent in shell modeling especially due to element formulation and geometry mismatch in the transition region seem to contribute to the modeling of all TPMS unit cell types, particularly for the calculation of the relative density values. It is demonstrated that TPMS lattice structures can induce mechanical bandgap behavior which can be controlled, or tuned depending upon cell type and volume fraction. This enables the designer to control the frequency range over which the bandgap exists and it can be tailored for vibration isolation purposes to damp the response of different mechanical excitations which is traditionally achieved by adjusting the overall mass or adding extra dampers to systems.

These results suggest that the combination of the different TPMS structures can be jointly used to meet the complex design requirements. Modeling methodologies presented in this study can be a strong enabler for the additive manufacturing technologies for the synthesis of size, shape, and material composition of TPMS structures.

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